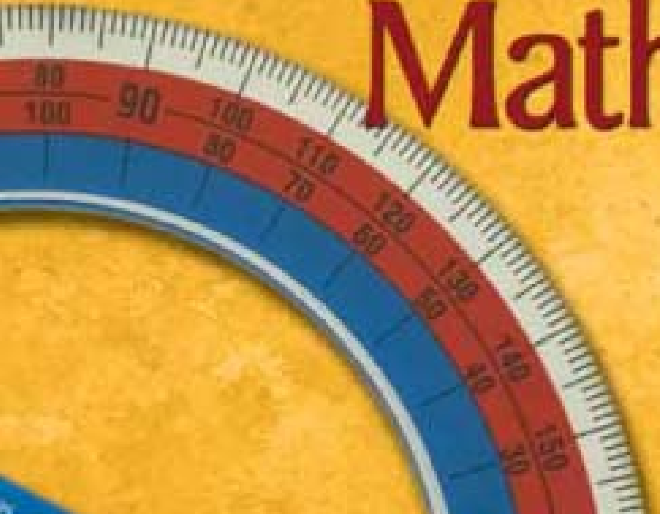
Professional *Development* for ***Successful*** Classrooms

Teaching Mathematics Today



Shelly Frei

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Shell Education
5301 Oceanus Drive
Huntington Beach, CA 92649-1030
http://www.shelleducation.com
ISBN 978-1-4258-0376-6

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Introduction: A Balanced Approach

Overview of Research

Teaching mathematics in today's diverse classrooms can be challenging, but it also provides teachers with many exciting opportunities to pass on life skills as well as mathematical knowledge. Mathematics is a subject that will directly affect every single student who enters the classroom. Proficiency in mathematics transcends simply succeeding in school or scoring well on state standardized tests. Mathematics teachers directly influence how students will approach problems and experiences in life: going to the grocery store, measuring walls to paint the house, borrowing money from a friend, cooking meals, driving a car, balancing checkbooks, paying taxes, buying houses, making investments, and so many other daily tasks.

There are many approaches to teaching mathematics. This professional development guide is built around a balanced approach to the instruction of mathematical concepts.

Common Approaches in Mathematics Instruction

Often teachers feel comfortable teaching the way they were taught. It is what they remember and what they know, so it becomes the way they teach, regardless of whether they believe it is the correct way to teach.

A common image of a "typical" mathematics classroom has the teacher standing at an overhead projector showing the equations and the formulas while the students take notes that will theoretically help them complete the assigned textbook problems. Many mathematics educators focus on skills and offer mostly procedural practice. Students learn the formulas and the procedures involved in the various mathematics disciplines. This form of instruction focuses on a lot of memorization and skill-and-drill practice. Teachers offer lecture type instruction and then students complete the pages in the texts during class time. Then they take home more practice worksheets for homework, with no further support for those who do not understand the mathematical procedures involved. This type of instruction is happening in elementary schools as well as middle schools and high schools.

Textbooks are also a large part of a "typical" mathematics classroom. However, the use of textbooks alone can create a reduced use of effective instructional time because the textbooks often lack relevant guidance regarding how to address different learning styles, engage students, integrate manipulatives, and differentiate instruction based on the many individualized learning needs that mathematics instructors see in their classrooms. Often textbooks are filled with distracting pictures and designs that do not add to the mathematical comprehension of the key concepts. The textbooks also include many more topics than

the average student can possibly learn in a school year. Teachers who lack guidance or experience may think they are supposed to open the textbook on day one and teach as far as they can by the end of the year. Therefore many educators are not teaching some of the important foundational concepts that students need in order to continue on to subsequent mathematics courses.

Another type of mathematics program leans more toward exploration of mathematical concepts through conceptual investigation. These programs are often very popular in elementary schools. Students use concrete materials, such as manipulatives, and participate in experiments and kinesthetic demonstrations that exhibit mathematical concepts. However, these types of lessons sometimes lack the connection bridging the "fun" activities to the actual mathematical concepts and abstract form of the problems. Sometimes this approach comes from a textbook-based program that offers little association between the learning activity with colorful pictures on one side of the textbook and the paper and pencil problems listed on the other side.

Often teachers are forced to follow strict district pacing charts or course outlines that delineate what concepts need to be covered during the school year. Because of the pressure teachers feel to cover all the topics, each topic is taught, practiced, and assessed and then the class moves on to the next topic. There is often little regard of whether students are actually able to truly learn and absorb each topic (Marzano, 2003). Consequently, only a subset of the students reaches a level of mastery of any given skill.

No one denies that there are indeed many students who succeed in these types of mathematics classrooms. They easily build on mathematical concepts and succeed in higher levels of mathematics courses. Yet there are also countless students who are struggling to achieve passing grades in their mathematics classes.

Many students are failing district and state mathematics assessments. If teachers only explain the rules and evaluate correct or incorrect answers, then the students come away with a limited view of mathematical expertise (Lampert, 1990).

While no one method of instruction has been proven as the single best way to teach mathematics, using research-based designs and procedures have helped educators recognize the best features for approaching learning goals in mathematics (Hiebert and Grouws, 2007). Today's mathematics teacher needs to develop a scope and sequence that integrates multiple opportunities for practice prior to assessment, and time to reteach and revisit those skills that students have not mastered after the assessment.

A Balanced Approach in Mathematics Instruction

Research has established that students need both procedural and conceptual knowledge in order to learn and understand mathematics (NCTM, 2000). Knowledge of the procedures and formulas are critical to overall proficiency in mathematics. Also, exploration of the concepts through concrete experiments and manual manipulation of mathematical concepts is vital to the overall understanding of the "why" in mathematics instruction.

Therefore, a balanced approach can link these two distinct approaches and offer connections that can lead students to higher proficiency and understanding of mathematical concepts. It is ineffective to emphasize a high degree of procedural proficiency without developing conceptual knowledge. It is necessary to provide focused instruction that moves the student from the concrete to the abstract and then to the application of the concept (Marzano, 2003; Sutton and Krueger, 2002). Focusing on only the conceptual knowledge is not enough to help students achieve in the classroom and in real-world situations.

Teachers integrate alternative teaching methods, manipulatives, and additional practice into standard classroom lectures with a balanced approach. Teachers are given assistance in how to plan instruction so that lessons will align with state standards and address students' needs. Teachers employ strategies that address different learning styles, engage students, and differentiate instruction. There are multiple opportunities for practice prior to assessment, and time to reteach and revisit those skills that students have not mastered after the assessment. When students are given sufficient practice, they can approach being able to use the newly learned skill in new situations with accuracy so that that skill will be retained (Sousa, 2006).

A balanced approach calls for a change in the classic role of the teacher in a mathematics classroom. Teachers become facilitators, helping to move students' understanding of a given concept from the concrete to the abstract, and finally to the conceptual application of mathematical concepts. Research by Fillroy and Rolano has shown that the transition from concrete models to the algebraic equations can be difficult for students to achieve (Kieran, 1999). Teachers need to be accessible to students and understand the process through which students need to progress in order for them to make the jump from the concrete phase to the conceptual phase. Following the concrete phase, students move to the abstract phase in which they learn the algorithm, using the understanding they gained from their experiences with the manipulatives. At this point, the conceptual understanding becomes important. It helps organize mathematical procedures and also helps students understand the appropriate procedures to apply in different situations (Kilpatrick, et al., 2001). Ultimately students need to move to a procedural fluency, which is "the ability to compute, calculate, and use rules and formulas correctly, quickly, and with assurance" (Dean and Florian, 2001). Without this, students will struggle to deepen

comprehension of mathematical ideas and the ability to solve mathematics problems (Kilpatrick, et al., 2001).

Research suggests that the traditional dichotomies of explaining the best categories for teaching skill efficiency and conceptual understanding are no longer helpful. The features of teaching that facilitate skill efficiency and conceptual understanding do not fall neatly into categories frequently used to contrast methods of teaching, such as expository versus discovery, direct instruction versus inquiry-based teaching, student-centered versus teacher-centered teaching, and traditional versus reform-based teaching (Hiebert and Grouws, 2007).

Therefore, the balanced approach suggested in *Teaching Mathematics Today* crosses beyond these common method labels to create a collection of great strategies, sample charts for recording classroom management information, and assessment information for teachers to easily and effectively use to meet students' needs. The suggestions in this book will assist teachers in developing their own research-based, best teaching strategies to address different learning styles, engage students, and differentiate instruction.

Components of a Balanced Approach in Mathematics Instruction

Standards-Based Instruction

The National Council of Teachers of Mathematics (NCTM) believes that all students should learn important mathematical concepts and processes with understanding (NCTM, 2000). In an effort to help teachers meet higher standards and the diverse needs of students, the NCTM Board of Directors has formed several documents designed as aids to anyone making decisions regarding mathematics education of pre-kindergarten through grade 12 students. Originally there were three separate documents. One focused

on curriculum, one on professional standards, and the last on assessment standards. These represented the first attempt to give extensive mathematics-related goals for the educational field, and were well received. Over time, these documents were revisited, reviewed, and revised. The result was that in 2000, NCTM published one document to encompass all the goals for teaching mathematics in education. It is called *Principles and Standards for School Mathematics*. It serves as the basis for many states' mathematics standards and as support for decisions regarding mathematics in schools and what should be taught at each grade level. The teaching strategies for a balanced approach to mathematics instruction that are described in *Teaching Mathematics Today* address each of the six principles within the revised NCTM document: equity, curriculum, teaching, learning, technology and assessment. By aligning with standards, districts can work toward the NCTM challenge that everyone deserves to understand mathematics. It is not only for a select few (NCTM, 2000).

Integrated Curriculum

> Rather than working on subjects in isolation from one another, studying reading apart from writing, and apart from math, science, social studies, and other curricular areas, children learn best when they are engaged in inquiries that involve using language to learn, and that naturally incorporate content from a variety of subject areas. (NCTE, 1993)

It is important for students to understand that education is not a series of compartmentalized subjects that have nothing to do with one another. Rather, students need to realize that learning is more like a rug, where all subjects are woven together to create a broad scope of understanding that is ultimately most useful when all the strands fit together.

Teachers of mathematics must put concepts into real-life context for students in order for them to understand the concepts and make them personal. "When mathematics evolves naturally from problem situations that have meaning to children and are regularly related to their environment, it becomes relevant and helps children link their knowledge to many kinds of situations" (NCTM, 1989). This real-life context, which is necessary for developing student understanding, comes from integrating other subjects into mathematics instruction.

Language skills are most commonly and easily integrated into mathematics instruction. From an early age, a student is exposed to literature and develops a level comfort using and discussing books. "Opportunities for discourse in both reading and mathematics instruction promote children's oral language skills as well as their ability to think and communicate mathematically" (Moyer, 2000). Literature also provides a familiar context through which mathematical concepts, problem solving, patterns, and data can be explored and understood. Students do not see the need to learn *about* mathematics until they can learn about real life *through* mathematics.

The nature of inquiry embedded in science and social studies lends itself to the use of mathematics as a tool for understanding and extension. Mathematical concepts such as data collection, comparison, and analysis; patterns; probability; and graphical representations can all be learned and understood using science and social studies topics.

Technology is an ever-growing and changing field in education. It is important for mathematics teachers to help students understand that technology can be used for more than playing games and text messaging friends. Mathematical concepts can be enhanced and explored through the use of the Internet, computer

software, graphing calculators, and other technology products.

Teaching Mathematics Today provides suggestions and strategies for teachers to integrate mathematics across the curriculum. It provides information about reaching all learners and broadening students' understanding of mathematical concepts.

Student Engagement

The strategies in *Teaching Mathematics Today* are geared toward engaging students and creating motivation for their learning processes. Cathy L. Seeley, 2004 president of the National Council of Teachers of Mathematics (NCTM), discussed in her message "Engagement as a Tool of Equity" how students' active engagement in their own learning impacts their achievements. She states, "Student engagement is perhaps our most important tool in our battle for equity." When students are actively motivated and busy reaching learning goals, they are also actively constructing knowledge and moving toward successful mastery of key concepts. The teacher is not the only indicator of student success in this model. Rather, the students have opportunities to have ownership and a greater understanding of the ideas and concepts that they are interacting with. When students are actively involved in writing, modeling, exploring, and discussing mathematics versus simply watching the teacher do these things, students are more likely to be successful (Seeley, 2004b). Using manipulatives, taking notes, presenting *PowerPoint* slide shows, and having students model problems are examples of strategies that actively engage students in the learning process. *Teaching Mathematics Today* presents various strategies for student engagement in mathematics lessons.

Differentiated Instruction

As students move from the concrete, to the abstract, to the application phase of learning, they are exposed to a concept or skill numerous times. Students should have multiple experiences with topics, allowing them to integrate the topics into their knowledge base (Marzano, 2003). However, not all students process the new information in the same ways or bring the same skill sets to the learning experience. Some students need extra time to process concepts and look at problems in different ways (Sutton and Krueger, 2002). Other students need further teaching or teaching presented in multiple ways. *Teaching Mathematics Today* provides charts, strategies, and tips for identifying individual student needs and how to differentiate instruction to meet those needs within the classroom.

Cooperative Learning

Cooperative learning tasks are encouraged and described in this book. Cooperative learning offers the opportunity for students to learn from one another (Sutton and Krueger, 2002). In addition, students can be actively involved in their success and assume responsibility for their own learning with the support of other students. All students can benefit from cooperative learning. It enables high-performing students to stretch their understanding of a concept and make new connections between material. "Results are quite promising for using peer-assisted learning with low-performing students . . ." (Gersten and Clarke, 2007a). These students benefit from seeing the material presented in multiple ways where they can be actively involved in learning mathematical concepts. *Teaching Mathematics Today* provides teachers with an understanding of how to incorporate this method of instruction into the concepts and lessons they are already teaching.

Problem Solving

Research shows that students who are not successfully mastering mathematical concepts tend to demonstrate slow or inaccurate retrieval of basic mathematical facts, lean toward impulsivity when solving problems, and have difficulty forming mental representations of mathematical concepts or keeping information in working memories (Gersten and Clarke, 2007b). One study found that children improved in overall mathematical proficiency when they were taught mathematics through problem-solving strategies. Not only were they achieving better test scores, but also increasing in the ability to communicate their understanding of the mathematical concepts orally and in writing. The conclusions followed that the problem-solving approach to mathematics showed students and teachers that the two were connected and that the problem-solving strategies helped with overall mathematical proficiency (Hartweg and Heisler, 2007).

Research has shown that real-life applied activities and problem-solving activities establish a contextual setting for many lessons, providing motivation and encouraging curiosity (Hiebert and Carpenter, 1992). Overall, the challenging and interesting tasks found in application problems help teachers engage students in learning (Seeley, 2004a). Integrating problem solving as one aspect of the curriculum follows the balanced approach of mathematics instruction.

Teaching Mathematics Today offers a step-by-step process to teach students problem-solving strategies. The set activity examples are meant to create independent, competent student problem solvers.

Guided Practice

In guided practice, teachers take a "we do" approach to help their students understand the concept being taught. The communication and interaction between the teacher

and student have to be more significant than just solving problems on a worksheet. "Practice does not make perfect, it makes permanent" (Sousa, 2006). Through the guided-practice methods encouraged in this book, teachers monitor students' early practice and make sure it is accurate. They provide timely feedback so that the skills are learned permanently and correctly. Guided practice helps reduce initial errors and informs students of the critical steps in applying new skills (Sousa, 2006). As already mentioned, *Teaching Mathematics Today* incorporates instructional strategies that balance procedural proficiency and conceptual understanding, while actively engaging students in practice experiences that are designed to deepen their understanding and connect their mathematical knowledge to real life.

Manipulatives, Games, and Calculators

Manipulatives are essential to helping students understand mathematical concepts. Using manipulatives regularly provides hands-on experience and helps students construct useful meanings for the mathematical concepts they are learning (Grouws and Cebulla, 2000).

The use of manipulatives has become common in the primary grades and has proven to be an effective tool for illustrating elementary mathematical concepts. When students use concrete objects to represent mathematical ideas, they learn to organize their thinking and reflect on concrete representations (Dean and Florian, 2001). These same tools can be very effective in middle and high school mathematics classrooms. For example, manipulatives, such as algebra tiles, have extended this physical representation into Algebra I and provide a basis for developing algebraic concepts (Sharp, 1995).

Students need ample opportunities to practice in order to be able to execute procedures automatically without con-

scious thought (Kilpatrick, et al., 2001). Playing games with the goal of reinforcing skills, rehearsing information, and building retention of mathematical concepts is one way to allow students the practice time necessary for a skill to be mastered. Students in today's classrooms are very motivated by entertainment. Mathematical games can pique their interests and give them a sense of fun while they are learning.

The use of graphing calculators can be helpful as well. According to research (1997) compiled by Dr. Bert Waits, cofounder of the developmental program Teaching with Technology, and Heidi Pomerantz, a professor of mathematics at Ohio State University, "graphing calculators can improve classroom dynamics, boost students' confidence levels, and promote understanding of mathematical concepts and functions." Further research shows that students who had access to calculators were better with mental calculations and estimations, as well as better able to solve real-life problems. Student achievement in general was higher (Kilpatrick, et al., 2001; Heller, et al., 2006). With increased use of graphing calculators during instruction, higher test scores were achieved even if students did not have access to the graphing calculators during the test (Heller, et al., 2006).

Teaching Mathematics Today discusses how manipulatives, games, and calculators serve as incredible tools for engaging students as well as addressing the needs of kinesthetic, visual, and English language learners. This book provides management techniques and strategies for using these materials in the classroom.

Vocabulary Development

Teaching Mathematics Today also provides teachers with resources for developing the specialized vocabulary necessary for mathematical-concept comprehension. Mathematical language is very precise compared

with the English used in common discourse. This makes the study of mathematical vocabulary different from most other content areas students study. Various vocabulary-development activities are available so that students, including English language learners, can truly understand the academic vocabulary that will help them unlock the mathematical concepts.

Intervention

Rather than waiting to find out which students will require intervention and additional instruction in order to pass the required mathematics classes, there is an increasing need for innovative programs that prepare students to comprehend mathematical concepts and fill in achievement gaps. "We must expect all of our students to learn mathematics well beyond what we previously expected. We need all students to be more proficient than in the past, and we need many more students to pursue careers based on mathematics and science" (Seeley, 2005). In order to reach these goals, mathematics programs are needed that offer foundational concepts with a balance of computational and procedural skills, conceptual comprehension, and problem-solving practice so that students can build on general mathematics proficiency. Effective intervention programs that prepare students for college and higher education should focus on readiness rather than just remediation (Oesterreich, 2000). Some of the most effective teaching practices suggested for low-achieving students as well as special education students are visual and graphic depictions of problems, systematic and explicit instruction, small-group instruction, student think-alouds, peer-assisted learning activities, and formative assessment data (Gersten and Clarke, 2007a). These teaching practices are infused in many of the strategies offered in this book.

Assessment and Data-Driven Instruction

Standards-based instruction begins with the goal of all students mastering the given curriculum with appropriate instruction, materials, and support. In order for this goal to be achieved, teachers must have a firm grasp of where students are in their process of learning a mathematical concept, what they need to accomplish to achieve mastery, and how they will reach the set goals (Wiliam, 2007). Teachers must then use formal and informal assessment strategies "minute by minute and day by day, to adjust their instruction to meet their students' learning needs" (Wiliam, 2007). Assessments provide teachers with the necessary data to understand which students are struggling in specific areas of the curriculum.

This book provides strategies and charts for formal and informal assessments, as well as ways to use data to drive further instruction within the classroom.

The National Council of Teachers of Mathematics Standards/Focal Points

> The Curriculum Focal Points are the most important mathematical topics for each grade level. They comprise related ideas, concepts, skills, and procedures that form the foundation for understanding and lasting learning (http://www.NCTM.org).

The National Council of Teachers of Mathematics (NCTM), one of the leading nationwide authorities in teaching mathematics, has been providing standards and suggestions for school mathematics courses for decades. The content standards found in the *Principles and Standards for School Mathematics* (NCTM, 2000) are used either directly or as a comparison standard for school districts across the nation. Many states refer to

this guide as a model of how to develop and cultivate mathematics comprehension for students who progress through each grade level toward graduation from high school. These content standards are divided by grade bands—prekindergarten through second grade, third through fifth grade, sixth through eighth grade, and ninth through twelfth grade. Each grade band encompasses the various disciplines of mathematics: number and operations; geometry; measurement; algebra; and data analysis, probability, and statistics.

Following the authoring of this comprehensive collection of standards, NCTM created the *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*. This document encompassed the major focus areas for which to provide emphasis for the included grade levels. The goal was to meet the needs of increasing accountability, students and teachers who move often, and the cost of continually developing mathematics curriculum. By pulling out only the most vital concepts that are necessary in that grade level, this document was created to describe baseline standards for student knowledge. In this document, there are descriptions of the mathematical concepts and skills, rather than lists of the goals, standards, objectives, and learning expectations, as are found in the standards document. The collection of descriptions was created to inspire teachers to discuss with one another the direction of the mathematics courses in the school. They were created to guide the formation of mathematics curriculum and to inspire the strategies and lesson plans used to teach mathematical concepts. "This work may assist in the creation and eventual development of new models for defining curriculum, organizing instruction, developing materials, and creating meaningful assessments that can help students learn critical mathematical skills, processes, and ways of thinking and can measure and communicate what students know about the mathematics that we expect them to learn" (NCTM, 2006).

How to Use This Book

Teaching Mathematics Today is meant to be a guide for mathematics teachers. The book is designed to span all the grade levels from kindergarten through grade 12, and also can be adapted for the various disciplines of mathematics.

- This book offers research-based explanations of the teaching strategies that are most critical and highly effective for mathematics teachers to include in teaching mathematical concepts.
- Each chapter focuses on a different aspect of teaching in a mathematics classroom.
- A school or mathematics department might choose to work through the entire book as they streamline and perfect their mathematics program.
- A school or mathematics department might choose to use this book as the basis for intervention programs being implemented.
- An individual teacher might choose to use the book to improve the effectiveness of mathematics instruction in his or her classroom.
- Teachers could keep the book as a resource to refer to for the specific areas as they relate to a mathematics classroom.
- New teachers can read the extensive explanations of the strategies and employ them in their lessons.
- Veteran and new teachers can read and apply the techniques that are described in the chapters to their current instruction of mathematical concepts.
- Teachers can learn extensively about meeting the different needs and offering access to core curriculum for struggling students and English language learners.

Post-Reading Reflection

1. What was your definition of a balanced approach to mathematics instruction at the beginning of this chapter?

2. Would you revise your initial definition? If so, how?

3. Reflect on two components of a balanced approach in mathematics instruction and explain why these are important for teachers to understand.

Planning Instruction

Team Building

One of the keys to an effective mathematics program is team building. A strong team lays the foundation for constructive curriculum planning, quality instructional courses, and meeting the students' total mathematics needs. A team should be put in place for educators and administrators to work smarter, not harder. Both small and large decisions have to be made regarding mathematics instruction. When a team is making those decisions, it is less likely that best teaching practices will be compromised and steps will be taken away from the overarching goal of standards-based instruction.

Depending on the school, district, county, and state, mathematics teams can take many forms and functions. But regardless of the type or function of the team, it is necessary to have administrative support and well-trained, enthusiastic teachers who are committed to increasing student achievement and implementing a well-structured curriculum.

Team Building in Mathematics Instruction

This type of team would most often be utilized at the school level and would consist of teachers within a particular grade level or mathematics department, or teacher representatives from all grade levels or mathematics departments. This type of team can make decisions for school-wide mathematics instruction or for a specific grade level's mathematics instruction.

When planning the curriculum for the school, the team needs to ensure that its curriculum aligns with the state standards and uses data to target students' needs and show academic growth. Effective implementation of the curriculum will be the driving force behind the overall mathematics program.

Prior to the start of implementing the mathematics program, the school mathematics planning team should discuss pedagogy and team consensus. It is recommended that the team meet regularly to review student progress and the timeline for all mathematical concepts. They should discuss upcoming lessons, reflect on best teaching practices, and discuss the students' needs. This team can evaluate the flow of content throughout the year or from one course to the next, and the consistency of policies across school mathematics courses.

In order for these meetings to be successful, the school mathematics planning team may want to make decisions regarding the following issues:

- how decisions will be made within the team
- how to make the students' needs the priority for all program-related decisions
- developing lesson plan ideas for upcoming concepts
- establishing curriculum timelines and adjustments based on students understandings

- how course placements can be flexible enough to meet changing student needs
- meeting to discuss assessment trends
- meeting to discuss the effectiveness of the decisions made on the school-wide mathematics program implementation
- how to implement formal and informal assessments in the classroom
- how to interpret data as a way to drive future instruction
- planning how to handle those students who miss classes
- planning how to handle those students who are frequently tardy to class
- if and how homework will be assigned
- if and how homework will be graded and reviewed
- deciding what administrative support is necessary to ensure that teachers are effectively implementing the school-wide mathematics program

Cross-Curricular Team Building

This type of team building would take place mostly at the secondary level. In general, elementary teachers teach across the content areas. They understand that best practices involve integrating the content areas and giving mathematics a function outside of direct mathematics instruction. However, with secondary teachers, this concept is not put into place as often. This type of team could be made up of department representatives across the content areas for each grade level or school-wide.

In order for meetings to be successful, teams may want to make decisions regarding the following issues:

- common curriculum strands between content areas
- which projects or activities can be completed for credit in multiple courses utilizing instructional skills necessary in each course
- effective ways to illustrate connections between content areas to students
- what administrative support is necessary to ensure that teachers are effectively implementing cross-curricular instruction

Vertical Team Building

This type of team would be used to discuss various topics of importance to mathematics instruction on a large scale. The purpose of this type of team would be to make vertical connections across mathematics curriculum and allow schools to keep in mind the broad spectrum of spiraling instruction. Representatives would then be able to report back to their home schools to update their colleagues and administrators on the topics discussed during the meeting. For small school districts, this team could be comprised of mathematics leaders from each school. In larger districts, it might be helpful to only include representatives from schools within each feeder pattern.

In order for meetings to be successful, teams may want to discuss the following issues:

- how concepts are taught in each grade level
- how to effectively teach particular concepts
- what curriculum resources are being used at each grade level

- what holes exist in student knowledge, how to bridge the gap in student knowledge, and how to correct instruction so that future classes of students do not have the same holes
- how to analyze the schools' mathematics data with attention to successes as well as changes or revisions that need to be made in order for students to be successful

Aligning Instruction with Mathematics Standards

The Need for Mathematics Standards

Standards-based instruction is the basis of education today, as well as the center of teacher and student accountability. There is a significant need for content standards in mathematics instruction (Dean and Florian, 2001). Standards-based instruction is one way to meet the critical need for all students to be on the forefront of worldwide technology, scientific and mathematical advances, trade, and development. Standards provide accountability so that students receive the necessary skills to continue their learning processes in further grade levels, and higher mathematics in college and beyond (Dean and Florian, 2001). Furthermore, the standards build on each other. For example, in a standards-based system, a fifth grade teacher can begin to instruct students with a reasonable expectation that the students come with some understanding of the mathematical concepts that build up to the fifth grade content standards in the lower grades. While there are always going to be students who struggle with the various concepts, standards-based instruction ensures that students have at least been introduced to each important mathematical concept for their grade levels and assessed on the levels of mastery of that concept.

While there is some variation, most states have established the required content that a student needs to know by the end of each grade level in each content area. Mathematics lessons should be based on the state content standards so that students can be equally and adequately prepared for future mathematics courses as well as any state-mandated mathematics assessments.

Dissecting the Mathematics Standards

Whether states have their own standards or use those based on NCTM standards, there is some level of grade separation. There is also distinction among the standards for each of the disciplines of mathematics.

NCTM uses the five content standards areas of numbers and operations, algebra, geometry, measurement, data analysis and probability; and the five process standards areas of problem solving, reasoning and proof, communication, connections, and representations (NCTM, 2000). Each of these areas encompasses goals that expand across all the grade levels. The goals lead to expectations that differ based on each grade-level band. These expectations further specify what the students in that grade-level band should be able to do. NCTM uses the grade bands Prek–2, 3–5, 6–8, and 9–12.

Some states choose to further distinguish in each individual grade, rather than using the grade bands. Regardless of how a state's standards are arranged, it is imperative for teachers to carefully study and understand the content standards in their states for their assigned disciplines and grade levels. It is also important to have some knowledge of the content standards of the grades surrounding their own in order to know where the students theoretically have been and where they need to go in mathematics development.

Once a teacher has a good understanding of the required standards, a plan or pacing chart needs to be created for the year that will help the teacher cover the necessary skills and concepts. It is best to do this in a team to ensure that all concepts are taught, proper time allotment given, differentiation strategies and creative lesson plans utilized, and revisions made based on student assessment are included.

It is no longer best teaching practice to open the adopted mathematics textbook to page one and work as far as possible by the end of the school year. There are often superfluous activities throughout the textbook, and many of the necessary state-mandated concepts may be covered in the back of the book, which the class might not even get through. Rather, a teacher must:

1. Examine the content standards.
2. Make a long-term, but flexible, plan to cover them.
3. Decide on a standard and design lessons that will align with the standard.
4. Finally, decide which portions of the mathematics textbook, supplementary materials, books, manipulatives, and lesson strategies will help the students learn the required mathematical concepts.

Any plan or pacing chart should allow for some flexibility. The teacher will continually assess students, formally and informally, to see if more time is needed to reach mastery of a content standard. Sometimes a teacher will need to spend more time on one foundational concept in order to begin to introduce a new concept that will build upon it. Also, assessments will help teachers learn when students already know certain concepts. Then, less time can be spent on these in order to provide more instructional time for unknown concepts.

Developing a Mathematics Curriculum Timeline

One resource that teachers can use for instructional planning is the *Timeline for Mathematics Curriculum* (page 35). This chart can help teachers ensure that all the necessary objectives are covered during any given mathematics course by offering long-range planning by the content standards.

Choosing Resources to Teach Mathematical Concepts

Because it is necessary to teach to the content standards and not "to the textbook," use of timelines reminds teachers to find the necessary resources to cover the required content standards. This can involve picking the appropriate lessons from the adopted mathematics textbooks. As a rule, following textbooks from page one to the final page is not the best use of instructional time. Often textbooks have lessons that are not required for given grade levels in a mathematics course. Sometimes textbook lessons are redundant or unnecessary based on student pre-assessments. Therefore, the timeline-planning process requires that teachers find supplementary mathematics resources in order to reach the objectives.

Sometimes teachers need to plan further practice games or manipulative activities in order to give students the required practice of a concept. Timelines are also useful for allowing flexibility in meeting student needs for enough time to learn a concept before moving on to a new concept. Teachers can use these resources, along with district or school-site pacing charts, to record progress toward teaching to the content standards. Teachers can also use timelines to demonstrate to administrators, parents, and other teachers that they are teaching the required standards.

First and foremost, teachers should give students diagnostic tests to find out what they already know. Based on student results, the teacher can make curriculum decisions and focus lessons that correlate with the items for which students did not demonstrate mastery. Then, teachers can complete the *Timeline for Mathematics Curriculum*.

Guidelines for Planning the Mathematics Curriculum Timeline

1. Always start by plotting out the content standards and mathematic objectives that need to be covered.
2. Choose the materials, textbooks, lessons, manipulatives, practice games, activities, and resources that will best address the standards and objectives.
3. Frequent assessment before, during, and after units of study will help the teacher to decide how much time is needed for each concept covered.
4. If applicable, use midyear benchmark/quarter tests as guides for when to introduce concepts during the year.

Directions for the Mathematics Curriculum Timeline

Write the day of the week or date in the first column. The second column is where you can record the content standard or curriculum objective being addressed. Write the name of the lesson in the third column and the mathematics resource or program title in the fourth column. Then, write the page numbers in the fifth column. In the sixth column, write which portions of the lesson will be covered. In the last column, write any suggested adaptations or notes that teachers may need for each lesson.

Timeline for Mathematics Curriculum

Course: ______________________________

Times and Days of Instruction: ______________________________

Day/ Date	Content Standard or Curriculum Objective	Lesson Title	Mathematics Resource	Pages	Lesson Components to Be Covered	Adaptations or Notes

Accelerating and Decelerating the Introduction of New Concepts

There is a great deal of pressure on teachers in mathematics classrooms today. Teachers are expected to educate their students in the appropriate content standards so that they *all* can demonstrate mastery. Sometimes when teachers are trying to teach the required mathematical concepts for their grade levels, they find that the students lack the appropriate foundational skills necessary to learn new information. Teachers need to arrange the year's required mathematical concepts so that the lessons and activities build upon each other in a straightforward way that most students will be able to follow.

Often, teachers are given strict pacing charts to uphold and are held accountable with the requirement of turning in frequent assessments of progress. Continuous academic progress for every student is a goal that all teachers have. But, inevitably, each classroom of students contains a wide array of abilities and the pressure to show continual growth for each student becomes increasingly challenging. Teachers may not feel comfortable modifying curriculum to meet the needs of struggling students. But, it may not be appropriate for every student to complete each lesson as it is laid out. Some students need extra time or practice and others do not. Additionally, teachers may likewise feel uncomfortable with the best ways to adjust the curriculum for gifted students. However, adjusting the pace of the curriculum is necessary when students vary in ability levels.

Guidelines for Accelerating and Decelerating the Introduction of New Concepts

With the use of the *Timeline for Mathematics Curriculum*, in addition to the tips and techniques mentioned in the differentiation section of this book, a teacher should feel well equipped to then make adjustments to lessons so

that new concepts and skills are being introduced at a rate appropriate to the student needs in the classroom.

Beginning placement assessments should be used to diagnostically show which concepts the students have already mastered and which concepts need to be mastered. The teacher can use this assessment information, taken at the beginning, middle, and end of the course, as well as throughout various units of study, to choose which lessons to cover and which lessons may not be as necessary.

A teacher might find that certain skills come easily to the majority of the students in the classroom. When this is true, the teacher can allow less time for the practice and application of those skills and choose instead to continue to the next planned lesson. Similarly, if a teacher finds that the concepts in a particular lesson are very challenging to the students, this teacher might allow more time for each component of a given lesson—the modeling, guided practice, independent practice, and application games and activities.

The teacher could also decelerate the pace of the lesson with more pair or group activities to further allow opportunity for mastery of the lesson concepts before moving on to a new lesson. If time allows, the teacher might offer an additional day of application activities before moving on to a new lesson.

Accelerating the Introduction of New Concepts

A teacher may adapt various lesson components for accelerating a single lesson in order to go through the concepts faster. These are options that the teacher would use if the students already understood much of the material. At times, it may even be appropriate to skip a particular lesson if the whole class has previously mastered the concept. This can be determined by the placement test.

The following sections show how a teacher can take various commonly included components of mathematics lessons and allow for acceleration of the lesson concepts.

Daily Problems to Solve—The teacher can display the problem(s) for students to solve on a transparency as they enter the room. This can also be offered as extra credit or work to complete when the practice problems are finished.

Vocabulary Activities—The teacher may ask the students to do a quick-write of the necessary terms at the beginning of the week in order to focus on only the unknown vocabulary.

Skill Practice—This component focuses more on independent practice and could be given as homework. The teacher can decide which problems need to be practiced in the classroom and which should be completed at home. It is not always necessary to complete every problem in order to show mastery of a certain concept.

Application Learning—The teacher may choose one day during a week for the games and extra activities in which students apply their concept learning. The teacher might also reduce the amount of time that is dedicated to these activities.

Assessment of the Objective—Quizzes can be given as take-home assessments. If a skill is assessed more than once, or if students have mastered a specific skill, the teacher may choose to eliminate a quiz.

Necessary Components	*Components Adaptable for Acceleration*
Vocabulary Activities—These activities are vital for English language learners. **Teaching the Objective—**Students must have a well-planned lesson with modeling and practice if they have not already demonstrated mastery of the lesson topic. **Skill Practice—**This should never be completely eliminated. Students always need to practice what they have learned in a lesson.	**Daily Problems to Solve—**Daily problems can be completed as an entry activity or for extra credit. **Vocabulary Activities—**Vocabulary activities can be completed as a group or can only focus on unknown terms. **Skill Practice—**Practice problems can be given as homework or fewer problems can be assigned in class. **Application Learning—**It is possible to reduce the time allotted each day for application activities or only complete them once-a-week. **Assessment of the Objective—**The students can complete take-home assessments.

Decelerating the Introduction of New Concepts

When students need more instructional time to master a concept, it is necessary to decelerate the pace of the curriculum. This is often necessary when students did not receive the proper amount of instruction on the skill in the previous year or they are new to the curriculum and have had minimal exposure to the skill. This can be determined by the placement test.

The following section shows how a teacher can take various commonly included components of mathematics lessons and allow for deceleration of the lesson concepts.

Daily Problems to Solve—Students can solve the daily problems and then work on a previous day's problem as review.

Vocabulary Activities—The teacher can plan multiple activities, extending into oral practice and written practice, to practice the lesson vocabulary.

Teaching the Objective—The teacher can offer multiple opportunities for cooperative learning activities as students work on guided practice of lesson concepts. The teacher can also offer more guided-practice problems to work on with partners.

Skill Practice—The teacher can give more problems. The teacher might choose to have the students work on the problems with partners in class before being asked to independently work on them at home.

Application Learning—The students can spend additional time on the application activities. They can work on application activities with partners, in small groups, or independently.

Assessment of the Objective—The teacher can use the same assessments as pre- and post-tests for lesson concepts.

Post-Reading Reflection

1. What teams exist for planning mathematics instruction in your grade level, school, or district, and what function do they serve? If there are no teams in place, which type of team would you like to see implemented, and why?

2. How do you plan your curriculum timeline each year? What will you add or modify on your current timeline based on the reading?

3. What types of modifications have you made in your classroom to accelerate or decelerate the pace of curriculum introduction?

Managing the Mathematics Classroom

Involving Parents and Students to Improve Program Efficacy

It is important to keep positive and open lines of communication with parents regarding the education of their children as they progress in understanding mathematical concepts (Seeley, 2004b). The following suggestions will contribute to success for all students in any mathematics course curriculum.

Parent Involvement

It is the teacher's responsibility to help parents understand specifically how they can assist their children in succeeding in a mathematics course. It would be very beneficial for all mathematics teachers to involve parents in the process early in the year and in positive ways.

1. Establish a good rapport with parents by using effective communication skills.
2. Enlist the help of parents as influential allies.
3. The *Parent Letter* (page 46) can be used or adapted as a resource for explaining the significance of succeeding in the mathematics course and the necessity for students to master mathematical concepts.
4. Show parents how they can be vital partners in encouraging their children to remain focused on learning the necessary concepts.
5. Make parent communication accessible by translating documents that are sent home if parents speak a language other than English.

Once positive contact has been made, parents will be more willing to help a teacher if behavior or academic problems arise. If learning and behavior problems are involved, they are best resolved when teachers and parents work together to examine the context of the problems and devise a solution. As soon as a problem occurs, it is important to call the parents to discuss courses of action to remedy the situation. When problems are discussed, always focus the conversation on the behavior and the specific ways in which the parent can assist the teacher in resolving the situation.

Documentation

Documentation is important. Therefore, the teacher should keep a log of each time the parents are contacted so that the teacher can refer to it if additional contact or administration involvement is needed. The log also serves as a measure of accountability for the teacher, students, parents, and administration. The log will reflect that the teacher notified parents as a preventative course of action prior to giving a failing grade or taking disciplinary action. The log can also show when the teacher contacted parents for positive praise and feedback. The *Teacher/Parent Contact Log* (page 49) can be used for this purpose. The teacher can quickly record the date of the contact in the first column and the time of the contact in the second column, check the topics that were discussed, and record to whom the teacher spoke. The teacher should add any final notes regarding action steps or follow-up in the last column. By keeping one log for each student, the teacher is able to quickly reference the reasons, frequency, and dates of contact.

Student Involvement

Creating a contract with the students is a democratic process that allows students to have ownership of their roles and their behavior in the classroom. Contracts can be developed after agreeing on the classroom rules and procedures with students. Furthermore, discussions can be initiated regarding both teacher and student expectations. Contracts can be used in the classroom setting or as a school-wide policy. The *Social Contract* (page 48) can be used for this purpose, or a school may develop its own contract. The *Student Letter* (page 47) can also be used to explain to students the significance of participating and succeeding in the mathematics course and the purposes for mastering the concepts. For younger students, the student letter and social contract can be adjusted to fit their grade levels and simplified for their reading levels and understanding.

Parent Letter

Dear Parents,

Your child is beginning a new mathematics course. The course is titled:

____

The program, texts, and resources that I will use are:

____

____

____

In this class, your child will work on mathematical concepts guided by the content standards. Your child will study these concepts using hands-on learning tools, participating in mathematics practice and application activities, and learning new vocabulary specific to the concepts. I will assess all students frequently to determine whether concepts need to be introduced, reviewed, or retaught.

Please encourage your child as he or she learns the foundational concepts, and help him or her establish a quiet place to study and finish any necessary homework. If you are not able to help your child with mathematics problems, please help your child write down which areas he or she is struggling with so that I can direct lessons toward those needs.

Feel free to inquire about your child's progress or let me know of any problems as they arise.

Please read the attached rules for the classroom, as well as the positive and negative consequences for behavior. Please sign this letter and return it.

Sincerely,

____

Teacher signature

____

Parent signature

Student Letter

Dear Student,

You are starting a new mathematics course. The course is titled:

____

The books and resources that you will use are:

____

____

____

This course will help you build on what you already know how to do in mathematics. You will participate in the lessons in order to learn the important mathematical ideas, skills, and vocabulary you will need to succeed in the future.

It is important for you to understand the concepts and the vocabulary so that you know how to solve a wide variety of problems in your math books, tests, future mathematics courses, and real-life situations.

You are responsible for letting me know if you are struggling with some of the concepts and need more practice. Also, please fill out the attached student contract. This will help me better understand the ways that you best learn in a classroom.

Please read the attached rules for the classroom, as well as the positive and negative consequences for behavior. Please sign this letter and return it.

Sincerely,

____

Teacher signature

____

Student signature

Social Contract

Student Name: ______________________________________

Teacher Name: ______________________________________

Class/Period/Section: ___________________

1. How would you like the teacher to treat you?

   ____

   ____

2. How would you like other students to treat you?

   ____

   ____

3. How do you think the teacher would like to be treated?

   ____

   ____

4. How do you want to resolve problems between you and the teacher?

   ____

   ____

5. Explain what the classroom needs to be like in order for you to learn best.

   ____

   ____

6. Explain what you can do to help make sure the classroom is a good learning environment.

   ____

   ____

Teacher/Parent Contact Log

Student Name: ______________________________ **Class/Period/Section:** ________________________

Phone Number: _____________ **Parent/Guardian Name:** ______________________________________

Topics: (Check all that apply.)

Date	Time	Type of Correspondence	Absences	Grades	Conduct	Tardies	Other	Spoke to	Comments/Notes

Lesson Delivery Strategies

The Affective Filter

The affective filter is best illustrated as a hidden "screen" that rises when stressful or emotional situations occur. A teacher cannot see that a student's "screen" is raised. Consequently, all the teaching for that day is blocked to the student because of his or her anxiety. Often distracting factors from home get in the way of the student's learning processes. However, at times, the problem may have to do with the mathematics class itself. Perhaps a student is anxious about his or her weak computation or problem-solving skills. Perhaps an English language learner finds it difficult to comprehend the teacher's fast manner of speaking. A student with special needs may be anxious about understanding the lesson concepts and become discouraged with the pace and materials of the class. These issues directly affect the students' affective filters.

When the affective filter is up, it is not easy to learn. Therefore, teachers need to work on lowering affective filters to teach the day's lessons by examining current practices that may inhibit students from learning the necessary content. The following tips may help teachers lower affective filters:

- Provide students with sentence frames for responding to questions or for explaining their thinking. This helps students express their thoughts using appropriate academic language.
- Give students sufficient time to think, process, and rehearse with others before asking a question or requiring an activity to be completed.
- Employ active lesson activities such as modeling, guided practice, step-by-step explanations, partner work, small-group activities, manipulative activities, and the application games.

- Utilize the strategies offered in the differentiation component of this book.

Every teacher needs to consider nonthreatening and encouraging ways to lower a student's affective filter in order to promote learning. Lowering the affective filter leads to increased ability to learn the planned content and apply learning toward success in the mathematics curriculum.

Questioning

In many mathematics classrooms across the nation, an observer might find that the teachers are often doing the majority of the talking and also asking the majority of the questions. However, students learning mathematics need to be actively thinking, engaging their minds, and solving real-life problems. This happens when teachers learn good questioning strategies.

In a mathematics classroom, questions should:

- keep students dynamically engaged in the lesson
- allow time for students to express and articulate ideas
- let students hear alternative solutions and reasoning from peers
- permit the teacher to check for understanding throughout the lesson
- allow time for the teacher to reassess the pace and direction of the lesson
- model for students the problem, solution, and reasoning process
- provide students with many opportunities to practice
- motivate their individual learning of mathematical concepts

In order to approach higher levels of thinking, teachers can look to Bloom's Taxonomy. In the last five decades, many teachers have used this as a sort of hierarchy of questions that advance from less to more complex levels of cognition. Benjamin Bloom, along with a group of educational psychologists, created a design of skills to organize levels of cognitive thinking. The plan was published in the book *Taxonomy of Educational Objectives, Handbook I: The Cognitive Domain*, (Bloom & Krathwohl, 1956) with what has become popularly recognized as Bloom's Taxonomy.

The following chart gives examples of effective questions using the five NCTM strands.

Strand	Goal of the Question	Examples
Number and Operations	Understand different ways numbers are represented and used in real life	• What is the . . . of . . . ? • Draw a picture to explain your answer.
Number and Operations	Understand number systems	• Explain what . . . are in . . . . • Estimate your total . . . and compare your answer.
Number and Operations	Understand the effects and relationships of operations	• How did you determine . . . ? • What are all the ways . . . ?
Number and Operations	Understand estimation in problem solving and computation	• What would happen if . . . ? • Is your answer valid, and why?
Number and Operations	Apply theories related to numbers	• Is your answer reasonable? • How different is . . . from . . . ?

Strand	Goal of the Question	Examples
Measurement	Compare, contrast, and convert systems of measurement	• What are all the ways . . . ? • Which unit of measurement would you use? Explain.
Measurement	Understand the need to measure quantities in the real world and use the measures to solve problems	• Explain what . . . is in . . . . • What relationship does . . . have to . . . ?
Measurement	Estimate measurements in real-world situations	• In what other situation could . . . ? • Is your answer reasonable?
Measurement	Understand use and selection of appropriate units of measurements and tools in real-world situations	• What are all the ways . . . ? • How would . . . have been different if it were smaller? larger? stronger?
Geometry and Spatial Sense	Visualize and illustrate ways in which shapes can be combined, subdivided, and changed	• What if . . . were . . . ? • Explain why you completed . . . the way you did.
Geometry and Spatial Sense	Describe, draw, identify, and analyze 2–D and 3–D shapes	• How is . . . like . . . ? • Compare and contrast . . . .
Geometry and Spatial Sense	Understand the use of coordinate geometry to locate objects in both two and three dimensions, and to describe objects algebraically	• What would happen if . . . ? • Can you construct a model that would change . . . ?

Strand	Goal of the Question	Examples
Algebraic Thinking	Use expressions, equations, inequalities, graphs, and formulas to represent and interpret situations	• What would happen if we took something away from . . . and replaced it with . . . ? • Which expression shows . . . ? Explain.
Algebraic Thinking	Describe, analyze and generalize a variety of patterns, relations, and functions	• Why is a . . . graph useful to show the kind of data given in a chart? • Can you guess my "rule" and explain?
Data Analysis and Probability	Understand and use the tools of data analysis for managing information	• What if . . . were . . . ? • What would happen if . . . ?
Data Analysis and Probability	Identify patterns and make predictions from an orderly display of data using concepts of probability and statistics	• How would you organize . . . to show . . . ? • What would result if . . . ?
Data Analysis and Probability	Use statistical methods to make inferences and valid arguments about real-world situations	• What conclusions can you interpret from your graph (data)? • Is your answer reasonable?

Chart adapted from Miami-Dade County Public Schools' Mathematics Question Task Cards (http://www.fldoe.org)

Effective Questioning Strategies

The common practices of "teacher asks question, students raise hands, teacher calls on one student to answer question" or "teacher calls on a student, teacher asks question, student answers correctly or incorrectly" simply do not provide all the benefits of good questioning in a mathematics classroom. In these situations, the teacher only hears an answer from one student while the rest of the students breathe sighs of relief that they are not being called on. Often, an observer notes that it is common for teachers to inadvertently call on the same students repeatedly. Students struggling with the content or with the language of instruction will be hesitant to raise their hands or even respond to the teacher's questions. Even their fears of being called on can keep them from processing the concepts involved in successfully answering the questions. Most importantly, these teaching practices do not allow all students to demonstrate knowledge and engage in mathematical reasoning and problem solving. The following questioning strategies can allow for each of the benefits of good mathematics classroom questioning listed above.

"Everyone Involved" Questioning Strategy

1. The teacher asks a question.
2. The teacher allows the students "think time" to process their answers to the question. If necessary, students are allowed to use paper to solve equations and process their own thinking.
3. The teacher directs the students to share answers with partners or small groups. The teacher encourages them to come to consensus on the correct answer and to be able to make a case for why their answer is correct.
4. The teacher calls on someone from one of the groups to give the group's answer.

The "Quick Check for Understanding" Cooperative-Questioning Strategy

1. The teacher asks a question and displays various responses on the board, one of which is correct.
2. The students work with partners to solve the problem and determine which answer on the board is correct.
3. The teacher has the partners display their answers.

The "Quick Check for Understanding" Individual-Questioning Strategy

1. The teacher asks a question.
2. The teacher allows the students "think time" to process their answers to the question. If necessary, students are allowed to use paper to solve equations and process their own thinking.
3. The students record their answers on a small whiteboard or piece of paper and hold them up so the teacher can see the responses.

The "Leveled" Questioning Strategy

One further issue to consider when questioning during a mathematics lesson is which level each student is on. A teacher should know the levels of mathematical understanding of each of his or her students. In addition, if any of the students are English language learners, the teacher should know each of their abilities to speak, listen to, read, and write in English. When the levels are properly assessed and known, teachers can use this for everyone's benefit in a lesson with the following strategies:

1. The teacher asks a question and calls on a student whose content-ability level or language acquisition level matches that of the question being asked.

Students of lower levels would be asked to answer lower-level questions and students of higher levels would be asked to answer higher-level questions.

2. Each question asked matches the level or is slightly above the level of the student asked.

The entire class benefits from this differentiation strategy as the students are working with each other to find answers and participating in the lesson according to their content knowledge or language levels.

Pacing a Mathematics Lesson

A well-paced mathematics lesson is very important. Effective teachers plan lessons that actively engage students during instructional time as they explore new mathematical concepts. This process begins with a well-planned and paced mathematics lesson that includes the following:

<u>Before</u> the lesson, time is not wasted due to student confusion or poor planning. The teacher can "jump into" the lesson concepts without delay.

- The content standard and/or lesson objectives are visible. These can be either in "official" language or in "student-friendly" language. Students should begin each lesson knowing what they are learning and why they are learning it.
- The teacher planned realistic lesson objectives that can be met by most of the students during the given class period or content block.
- The teacher gathered and prepared lesson materials for all the lesson activities before the students entered the classroom.
- Materials are suitable for the age and ability levels, or modified and adapted so that they will be comprehensible.

<u>During</u> the lesson, the teacher incorporates effective lesson components where students are on task and engaged in learning new concepts.

- The lesson begins with anticipatory activities that connect students' prior knowledge to the mathematical concepts they will be learning. The teacher begins to create enthusiasm and helps students understand how prior lesson concepts will build on what they will learn in the new lesson. Important vocabulary can be introduced.

- The teacher uses direct instruction and teacher modeling to introduce new mathematical concepts.

- The teacher balances teacher-directed activities, note taking, guided practice, and interactive activities as students learn about new concepts.

- The teacher continually checks for student understanding during the lesson. The teacher adjusts the pace and the length of time necessary for the various lesson components. By checking for understanding, the teacher can be flexible with the time needed for reviewing or reteaching concepts.

- The students get the chance to practice skills and apply mathematical reasoning. The teacher guides student practice.

- Students are given the chance to practice with peers and to use the four domains of language—listening, speaking, reading, and writing—as they practice new mathematical concepts.

At the end of the lesson, the teacher brings closure and ties key mathematical concepts to the overall foundation that is built upon as students approach mastery in the specific content standards. The teacher makes decisions on whether students can continue with new concepts or whether more review, practice, or instruction is necessary.

- Independent-practice activities, informal or formal assessment activities, or projects that match the planned objective, as well as the guided practice that the students engaged in during the lesson are used to determine the students' understanding of the lesson.

Common Mathematics Classroom Management Issues

The Physical Environment

While this may seem like a petty consideration, the physical environment in a classroom is often the outcome of a teacher's outlook and educational pedagogy. The classroom setup also has an effect on the way students, parents, and others feel as they enter the room. Teachers need to examine the goals they have for student learning and then set out to arrange the classroom in a way that will facilitate the types of learning activities that will be most motivational for student success.

Walk through the classroom you have set up. Consider these questions.

- Can students easily access their desks, materials, textbooks, and manipulatives? How will they get paper, pencils, or other supplies before or during mathematics lessons?
- Is the classroom set up for teacher lecture only, or is there some allowance for cooperative learning and interactive activities?
- Does every desk have easy visibility of the board or overhead projector?
- Are the walls cluttered? Can students find information they need to help them review major mathematical concepts?
- Are the walls bare? Is there any support as students try to remember key vocabulary, formulas, and ideas specific to mathematics?
- What colors have been chosen in the classroom? What general emotion do they convey?
- Is there a place to clearly display classroom rules, consequences (positive and negative), and daily lesson objectives?
- Is there a place where student work and efforts are celebrated?
- Is the general look of the classroom cluttered and prone to distracting students, or clean and organized?
- Is the physical space easily adaptable for direct classroom lecturing, cooperative activities, centers or group activities, and quiet independent practice of skills?

Interactive Learning Activities

Many teachers hesitate to use interactive learning activities because they fear the chaos that may ensue. Others love to throw groups together to work and then sit back at their desks and use the time to correct papers, without monitoring the work that is being done. These are two extremes to approaching interactive learning in a mathematics classroom. Neither of them is recommended.

The following guidelines will help teachers ensure successfully incorporating interactive learning activities into a mathematics lesson.

1. The planning and preparation for interactive activities needs to be done before the students enter the classroom. All materials need to be assembled and ready to use.
2. Plan a simple lesson for the first time that an interactive activity is done in class. Model and role-play the correct and incorrect procedures expected of the students for this type of activity. As the students become more familiar with the procedures of the activity, more in-depth mathematics content can be introduced.
3. The teacher must tell or remind the students of the following every time that they will participate in an interactive activity:
    - the activity's objective and purpose
    - the expectations for student conduct
    - the consequences for not meeting the expectations
    - the procedures for the interactive activity

4. Follow through on the consequences. The students will soon learn that they enjoy interactive activities more than straight lecturing and will probably be more apt to follow the conduct expectations.

5. The teacher is the most active person in the room during an interactive activity. The teacher is constantly walking around the room in close proximity to all the students, monitoring the responses and the student work, answering questions, clearing up any misconceptions that arise, and giving feedback.

6. If the first time is not successful, try again with that *same* activity on another day. Go through all of the above reminders again and give them another chance. It takes time for students to become familiar with the procedures of a new activity.

Mathematics Classroom Procedures

Every successful classroom needs established rules, procedures, and consequences. Every teacher has a different way to effectively set these up in the classroom. Classroom rules should be established early in the school year and clearly posted for students. Consequences, both negative and positive, are also made clear to students early in the school year. However, what often is not so clear are the procedures. The block of time designated for mathematics instruction each day (or each mathematics period) has its own procedures. Every mathematics teacher needs to make decisions on the procedures specific to the mathematics instructional time and make the expectations clear to students.

- What will students do when they are finished with an assignment during independent practice?
- Are students allowed out of their seats for any purpose during independent practice or during cooperative activities?
- What noise level is acceptable for the classroom or a particular type of activity?
- How will students get paper or sharpened pencils if they need them?
- How will the teacher distribute manipulatives or calculators? How will these things be collected after their use?
- Is restroom use or drinking water from a water fountain permitted during instructional time?
- What should a student do if he or she needs assistance solving a problem?
- During group work, may a group discuss ideas and questions with other groups?
- How will homework be assigned, collected, and graded?
- How will students access correct answers to homework or independent-practice problems?

Maximizing Instructional Minutes

In today's educational environment of high accountability and high stakes, it is the teacher's responsibility to maximize every possible minute available for student learning. Teachers have to throw out any superfluous activities that do not directly relate to the content standards and objectives, even if they have always used the extra activities.

Be Prepared

The first way to achieve the goal of maximizing instructional time is for the teacher to be well prepared. It is also helpful to be well acquainted with the content standards and the course concepts that need to be mastered by the end of the year. Talk to teachers in the year ahead to see what skills they feel are most important for students to have mastered for the curriculum they will cover in the following year.

Teachers should spend time creating their daily lesson plans so that instructional time is not wasted with preparation during class time or with the implementation of unpracticed procedures.

For each day's lesson:

- Choose the content standard.
- Plan the lesson objectives to be covered in one lesson period.
- Plan lesson activities—pre-assessment, review, instruction, modeling, guided practice, checking for understanding, independent practice, cooperative practice activities, application activities, and assessment of lesson objective.
- Gather the materials and resources.
- Give students the big picture by writing the lesson's goals clearly for students to see.

1. If the teacher has **the lesson plan ready and the materials prepared** before the students enter the room, there is less wasted time in which the students have to wait for the teacher to gather the day's instructional resources.
2. In a mathematics lesson that involves multiple components (pre-assessment, review, instruction, modeling, guided practice, checking for understanding, independent practice, cooperative practice activities, application activities, assessment of lesson objective), the teacher should **look over the various components beforehand** to see if any additional materials are needed.
3. The teacher should **plan for extra activities** if there is extra time at the end of the class period or for students who finish early.
4. The **answers to any lesson problems should be solved and accessible** before the teacher begins the lesson. This will help the teacher quickly assess the student's answers in order to pinpoint those who still need help.

Utilizing Additional Instructional Time

Sometimes a lesson goes faster than the teacher anticipated. The teacher should always have a few activities planned to make the most of additional instructional time in the classroom.

- **Explain the Concept/Summarize the Lesson—** Have the students work with partners to explain the day's concept to one another. This gives additional practice and allows students the chance to rehearse the academic vocabulary essential to the concept. Most students understand a concept better once they have had to explain it to someone else. This is very beneficial for English language learners and struggling students.

- **Personal Agendas—**Each student has a chart with a list of activities that are appropriate for him or her to complete independently. These activities can be based on skills needed for remediation or skills for acceleration, depending on the student. When a student has completed an independent activity on his or her personal agenda, he or she must obtain the teacher's initials next to the activity's name and description on the chart in order to move to a new activity.
- **Solve to Earn a Pass—**Ask each student to solve a simple problem orally, or give an explanation or definition as the "pass" to get into line in order to leave the class.
- **Numbered Heads Together—**Have the students quickly form "Numbered Heads Together" groups by getting into groups of four and numbering off one to four. They put their heads together to solve a problem given by the teacher that is based on material covered on a previous day. They must make sure everyone understands the answer, because the teacher then rolls a die or spins a spinner to determine which number of student from each group will give the group's answer.
- **Standardized Test Preparation—**Give each group of four students a set of answer cards labeled A, B, C, and D. Then the teacher can give them test preparation-type problems to solve. Each possible answer is labeled A, B, C, or D. It is easiest to have these questions prewritten on a transparency. In their groups, each student solves the problem individually. The group then discusses their solutions in order to come to consensus on the answer. The teacher then directs each group to hold up the answer card it chose. The whole group is responsible for knowing the answer as well as the explanation for why the other possible answers do not work.

- **Problem Solving Journal—**Have students write about their understandings of vocabulary, concepts, or procedures in journals.
- **Real-life Application—**Students practice real-life application activities and games from previous lessons in order to further practice with mastered or nearly mastered concepts.

Managing Classroom Time with Efficient Transitions

In a typical mathematics lesson, the teacher may direct the students through various modes of activity. A lesson may include whole-class discussion, pair work, small-group discussion, cooperative activities, games and application practice of skills, and independent work.

During a given lesson, the teacher will want to maximize instructional time of mathematical concepts and minimize transitions from one lesson activity to the next. Without clear expectations, rules, and procedures, transitions can cause confusion that takes away from learning.

Transition time is commonly the time when behavior problems arise. This is especially true if the proper procedures have not been taught and regularly practiced. Every teacher should strive to efficiently establish transition procedures. When transition time in a mathematics classroom is limited, time for learning and applying concepts and skills are increased. Taking extra time at the beginning of the year to model and role-play correct and incorrect transitions will save instructional time later in the year. This is true even for older students.

When initially practicing transitions, as well as throughout the year, the teacher needs to prepare the students to make effortless transitions.

1. Be **clear about expectations**. Tell the students what they will be doing and check for understanding before allowing them to transition to a new activity.
2. Rehearse transitions from various types of classroom activities often. The students will need **ample opportunities to practice**.
3. Choose **an auditory or visual signal** to help students know when to change activities. When using a signal for transitions, the students need time to think before they are required to act. The teacher should give the signal, which instructs them to be silent, inform them of the expectations, and then give the signal again. That gives them permission to move to the next activity.

Possible Signals to Use During Transitions:

- turn lights on and off quickly
- squeaky toy
- classroom chant
- counting backwards
- hold hand up in air
- wind chimes or xylophone
- thumbs up
- train whistle
- rain stick
- bell or gong

Post-Reading Reflection

1. Reflect on your teaching style. What teaching practices or behaviors do you have that may cause students to put up their affective filters? How can you work to change those behaviors or teaching practices?

2. Choose a lesson that you enjoy teaching. Create at least five questions using the guidelines and strategies provided for effective questioning. Make sure that you have at least one question for each level of student in your classroom.

3. Choose an interactive activity you have used in the past or create a new activity for a concept you are teaching this year. Make a list of the procedural steps, materials, and transition techniques you will use to allow the activity to run smoothly.

Differentiating Instruction

The Importance of Differentiated Instruction

Not too long ago, it was thought that students in the same grade level sharing the same class learned in similar ways. Today, it is clear that this viewpoint is wrong. Students have different learning styles, come from different cultures, have different levels of language abilities, experience a variety of emotions, and have varied interests. Because students differ in academic readiness, teachers have realized that they must differentiate their teaching to better meet these diverse needs.

Differentiation has many faces depending on the particular students and teachers involved, the outcomes

of these learners, and the structure of the classroom environment (Pettig, 2000). Differentiation encompasses what is taught, how it is taught, and the products students create to show what they have learned. These three categories are often referred to as content, process, and product. Teachers should differentiate content, process, and product according to students' readiness, their learning styles, and their interests (National Research Council, 1990). If a learning experience matches closely with a student's skills and prior knowledge (readiness), he or she will learn better. Creating assignments that allow students to complete work according to their preferences (learning styles) will help learning experiences become more meaningful. If a topic sparks excitement in the students (interests), then they will become more involved in learning and will better remember what they learned. To make the mathematics lesson activities most effective, teachers should differentiate the lessons. Not all students need to be engaged in exactly the same activity at exactly the same time.

Differentiated Learning

All students learn differently and struggle with different mathematical concepts. Even the level of struggling that a student experiences varies. Because of this, many of the same researchers who created the Reading First initiative developed a system of identification known as Response to Intervention (RTI). The RTI model supports the idea that teachers should look for curricular intervention designed to bring a child back up to speed as soon as he or she begins having problems. "RTI has the potential then to allow disabilities to be identified and defined based on the response a child has to the interventions that are tried" (Cruey, 2006). Depending on the levels of difficulty they are having with the mathematics curriculum, students are classified as Tier I, II, or III.

Teachers who differentiate instruction need to:

- Allow an appropriate amount of time for planning and preparation of instruction and activities.
- Assess students often and use the results to guide instruction and appropriate intervention strategies, if necessary.
- Make specific plans to meet varied student needs and communicate those plans clearly to parents.
- Utilize many flexible grouping strategies throughout classroom activities.
- Vary the types of activities and modes of instruction used in the classroom to meet the learning needs of all students.
- Continually reflect on personal teaching strategies and modify the methods of mathematics instruction if students are not responding positively to the delivery methods.
- Ask for help from specialists, other mathematics teachers, parents, administrators, and anyone else who can help respond to student needs.

Differentiation by Specific Need

Tier I Students

Tier I students are generally making good progress towards the standards, but may be experiencing temporary or minor difficulties. These students may struggle only in a few of the overall areas of mathematical concepts. They usually benefit from peer work and parental involvement. They would also benefit from confidence boosters when they are succeeding. Although they are moving ahead, any problems that do arise should be diagnosed and addressed quickly in order to ensure that these students continue to succeed and do not fall behind.

Tier II Students

Tier II students may be one or two standard deviations below the mean on standardized tests. These students are struggling in various areas and these struggles are affecting their overall success in a mathematics classroom. These students can usually respond to in-class differentiation strategies and do not often need the help of student study teams.

Tier III Students

Tier III students are seriously at risk of failing to meet the standards as indicated by their extremely and chronically low performance on one or more measures of the standardized test. These students are often the ones who are being analyzed by some type of in-house student assistance team in order to look for overall interventions and solutions. In the classroom, these would be the students who are having difficulties in most of the assignments and failing most of the assessments.

English Language Learners

Students who are **English language learners** are learning concepts and language simultaneously. They need to have context added to the language. While they may have acquired social language skills, the language of mathematics is very academic in nature. This is one of the most important keys to success with English language learners, as these students acquire the necessary vocabulary for greater comprehension of the course content.

Below-Grade-Level Students

The **below-grade-level** students will probably need the concepts to be made more concrete for them. They may need more work with manipulatives and application games. Furthermore, students who are struggling may

feel discouraged by the number of practice problems on a typical practice page. By giving them extra support and understanding, these students will feel more secure and have greater success.

Above-Grade-Level Students

Many of the objectives covered in a mathematics class are new to the students. All students need a firm foundation in the core knowledge of the curriculum. Even **above-grade-level** students may not know much of this information before a lesson begins. However, the activities and end products can be adapted in order to be appropriate to those students' individual levels. High-achieving students can create a different end product in order to demonstrate mastery or be given different amounts of work if mastery of a concept comes quickly.

Differentiation Strategies

Strategies for English Language Learners	Strategies for Below-Grade-Level Students	Strategies for Above-Grade-Level Students
• Always do the vocabulary development component and allow extra practice to apply and use the vocabulary with the concepts. • Allow more time to simultaneously process the language and the content. • Use visual displays, illustrations, and kinesthetic activities. • Offer notes that are partially filled in so that students can focus on necessary information. • Start with concrete examples and use manipulatives. • Reduce the total number of problems. • Plan for oral rehearsal with partners of the academic language behind the mathematical concepts. • Evaluate the use of word problems. Read them aloud and emphasize key words that indicate procedural action. • Allow for partner work.	• Allow partner work for oral rehearsal of solutions. • Allocate extra time for teacher-guided practice. • Model often, showing them step-by-step how to solve the problems. • Allow for kinesthetic activities where they organize the step-by-step processes on flash cards before they actually use the information to solve problems. • Shorten the number of practice problems in a single work session. • Have easy-to-follow notes of the most important procedural information already made up for these students to add to. • Use activities centered on students' interests. • Reference the chart on the next page, depending on the degree of difficulty a student is having.	• Offer accelerated processing activities or allow for these students to skip practice activities that they already have mastered. • Shorten the number of practice problems. • Assign only the most difficult problems. • Assign step-by-step explanations of the solution process. • Have the students create notes and procedural steps to guide the rest of the class. • Request oral presentations of the concepts, which will benefit all students in the classroom. • Have students create games for practicing concepts and skills.

The chart on this page gives suggestions for differentiation based on the degree to which a student is struggling with mathematical concepts.

Strategies for Tier 1 Students	Strategies for Tier II Students	Strategies for Tier III Students
• Use assessments to identify the areas in which these students are not at mastery. • These students would benefit from pair work in which sometimes a student is the teacher (in the areas in which this student excels) and sometimes the student is the peer learner. • Reteach concepts in a different way. • Allow small groups to study concepts together. • If possible, ask for parental involvement in keeping these students on task in assignments. • Offer additional practice in the areas in which this student is experiencing difficulty. • Allow partner work for students to check the work and build confidence that they are on the right track.	• Use assessments to identify the areas in which these students are not at mastery. • Follow the suggestions for below-grade-level students listed on the previous page. • These students would benefit from a before-school or after-school intervention program. • Reteach concepts in a different way. • Offer extra practice in struggling areas with study groups or peers. • Allow time for peer tutoring. • Extended mathematics instruction would benefit these students.	• These students require extra intervention programs. • Consider before-school or after-school programs, or extended mathematics periods to combat the risk of failure. • The school's student assistance team can determine if students in this category might need testing for special education needs and an individualized education program (IEP) or modifications during assessment.

Grouping Strategies for the Classroom

When considering each day of mathematics lesson activities, the teacher needs to decide the best ways to group the students. The teacher should consider whether the content can be best learned and practiced in:

- a whole-group setting
- small groups
- partners
- individually

Small-Group Learning

With the small-group formation, the teacher needs to consider whether to place the students together with similar abilities (homogeneous groups) or with varying abilities (heterogeneous groups). It is generally agreed that it is not effective to have the students work in the same groups for the entire program. Rather, **flexible grouping**, where students are grouped differently for various activities and assignments throughout the day, allows students to get different feedback and support from classmates (Radencich & McKay, 1995). It also keeps students from singling out or labeling the "slow group" or the "smart group."

Paired Learning/Working with Partners

Giving students a chance to work with partners allows them to have more time to process new content and practice applying new knowledge in less stressful situations.

There are many instances when paired learning is a useful grouping strategy.

- Often, when manipulatives are being used, two students can sit side by side and share materials so that each has an opportunity to manipulate the materials.

- When students need to rehearse information about concepts they have learned or solve a problem together, they can discuss the steps necessary before sharing the work with the class.
- English language learners frequently need to practice using the necessary vocabulary for each mathematical concept. They need time to say the vocabulary words, use them in context, and get immediate feedback about what they said.

Independent Work

Working cooperatively with others is instructionally sound, but it should not be viewed as the only way to effectively teach students. They also need opportunities to explore on their own and to examine, question, and hypothesize about their own learning. Learning on one's own develops students' confidence and natural curiosity. Students will have higher success with independent work if it directly relates to the skills they were working on during guided-practice activities. When students are engaged in independent work, the primary goal of the teacher is to monitor their processes. How closely a teacher monitors students' work during independent practice is usually a good indication of how well the students will actually retain the information. This is also a good time for teachers to give lower-ability students one-on-one or small-group instruction.

Cooperative Learning

At times it is appropriate for teachers to organize students into small groups in order to accomplish an academic task. Within each group, the students are given instructions for completing a task. This is called cooperative learning. It is different from common "group work" in which students just work in a group together, without individual accountability and roles. In order for

the group to be considered "cooperative," each student needs to be assigned a role. Therefore, no matter what is being asked of them, all students are participating.

To make cooperative grouping effective, teachers need to consider the students' achievement levels. The students can be grouped into **heterogeneous groups** in which students' ability levels are varied, **homogeneous groups** in which students have the same ability levels, or **flexogeneous groups** that combine both heterogeneous and homogenous groups within one lesson. Within a mathematics classroom, all three types of grouping techniques can be used. Teachers should evaluate lessons to determine which technique will work best for each individual lesson.

Some teachers worry that not all students will participate in cooperative learning groups. There are steps that teachers can take in order to ensure cooperative participation.

1. **Teach appropriate social skills** to students by demonstrating these behaviors and role-playing with students.
2. **Plan activities** where all students are
   - working concurrently,
   - positively encouraged to help each other,
   - given roles that are codependent on the other students, and
   - individually accountable for the work that is turned in.
3. **Use a student-led group evaluation** where students can formally or informally evaluate
   - how they worked together,
   - what their group did well, and
   - what their group could do to improve.

Choosing a Grouping Strategy

The chart below analyzes some of the most effective grouping strategies to employ based on the teacher's goal for instruction. Please note that a teacher could choose one grouping strategy for a particular goal in the lesson or use multiple strategies to effectively reach the single goal.

Goal for Instruction	Grouping Strategy to Employ
Deliver lesson instruction to all students.	Whole-group instruction
Check for understanding during lesson.	Pair, small group, or independent activity
Have students think about and practice giving an answer to a question before responding to teacher.	Pair sharing activity
Have students practice a concept.	Pair, small group, or independent activity
Offer practice for using academic language of mathematic vocabulary.	Pair sharing activity or small-group activity
Have students work in groups while teacher identifies major needs for re-teaching.	Homogenous small groups
Have students teach and learn from each other while practicing a concept.	Heterogeneous small groups
Have students orally apply a concept.	Pair or small-group activity
Use manipulatives.	Pair or small-group activity
Have students apply a concept.	Pair, small-group, or independent activity
Offer self-supporting practice time.	Independent activity
Have students demonstrate mastery of a lesson concept.	Independent assessment

Group Activities

There are a variety of group activities that engage learners and allow teachers to differentiate the pacing, practice work, and actual instruction during lessons.

- Students are often very social creatures. Cooperative grouping caters to this social side of their personalities. It also gives them opportunities to practice the academic vocabulary surrounding the concepts. Furthermore, group activities often boost the confidence and risk-taking that will later affect individual student work.
- Physical games help kinesthetic learners remember the key concepts better and get students out of their seats.
- Playing board games or card games challenges the students and encourages learning. Playing games as a whole class on the board or overhead can serve as a form of informal assessment as the teacher checks for student understanding. These games are fun and educational and also help ensure that all students are successful.
- Large-group games provide a safe environment for lower-level students to take risks. These students are supported by their teammates and can stretch their thinking outside of their comfort zones.
- Creating a game based on the concept of the teacher "competing" against the class allows students to participate and feel good about beating the teacher. This game works well using hangman as a model and adapting the skills the students are working on to fit that model. If the class answers the question or solves the problem before the hangman is complete, they get a point. If they can't answer the question or solve the problem, the teacher gets a point. The class always wins.

Many teachers cringe at the supposed chaos of group activities. However, teachers can follow these guidelines to help structure group activities and promote active learning.

1. Explain the **proper expectations and group-work rules.** These activities can provide the necessary practice and processing time for comprehension of lesson concepts.
2. Students need **practice with the procedures** involved in group-work activities.
3. It is important to **provide the appropriate time limit** for students to work in small groups or pairs to complete problem sets.
4. Group work allows **students to serve as peer teachers** by helping one another learn the concepts. Sometimes the teacher can even learn from the students by noting how the students explain the ideas to one another.
5. Allow the **students to evaluate** how effectively the group work aided their comprehension of the mathematical concepts.

If a group activity did not work the way the teacher intended, it should be completed again on the following day. This may help students understand the rules, expectations, and consequences more clearly. Often, students are excited about the chance to work in groups and will cooperate and follow directions when they fully understand the expectations and consequences.

Post-Reading Reflection

1. How are differentiation strategies for English language learners similar to differentiation strategies for below-grade-level students?

2. List three ways you can differentiate mathematics instruction for students in your class who are performing above grade level.

3. Choose one lesson that you plan to teach this year. Decide on a grouping strategy that will enhance the lesson and provide meaningful learning opportunities for all students.

Developing Mathematical Vocabulary

Specialized Mathematical Vocabulary for All Learners

In mathematics, vocabulary is highly specialized. These words are often not encountered in everyday life. Therefore, all students need an explicit introduction and explanation of these vocabulary words in order to be able to apply them to their understanding of mathematical concepts. The task is even more difficult for English language learners. These vocabulary words are not typically the words that English language learners will learn during their structured English Language Development class period. Therefore, it is up to the content teacher

who teaches mathematics to make certain that English language learners learn the necessary vocabulary in order to achieve comprehension of mathematical concepts and curriculum.

Furthermore, the different areas of mathematics (e.g., number sense and mathematical reasoning) and the various disciplines (e.g., algebra, trigonometry, geometry, and calculus) have different compilations of specialized vocabulary words. Sometimes there is overlap with words across mathematical areas and disciplines, but often there are new words that are specific to just one mathematical area or discipline. It is vital to understand the vocabulary for a specific discipline in mathematics because this knowledge aids in access to the core curriculum. All students are required to demonstrate mastery of the concepts, and this will only be possible if they first achieve understanding of the vocabulary words that explain, describe, and facilitate each of the mathematical concepts.

Vocabulary Development for English Language Learners

It is important for all students learning mathematics to be familiar with the specialized vocabulary embedded within the practice and application of the concepts. English language learners especially need consistent, structured instruction in learning English (English Language Development). They also need well-planned, sheltered instruction throughout content lessons and effective activities to develop mathematics vocabulary (Dean and Florian, 2001).

It is not enough to give the students a list of words and have them look up the definitions in dictionaries or textbook glossaries. Students who are struggling with learning a language are not going to find the process easier by simply being given more words to sort through. What

English language learners need are context-embedded lesson activities that acquaint them with the necessary words for comprehension of the content and allow them to practice the use of the words in activities that span listening, speaking, reading, and writing actions.

Mathematics teachers need to be cognizant of the language difficulties students have who are learning English. Many mathematics teachers believe English instruction is the job of the English teacher. However, the English teacher is not focusing on the specialized mathematics vocabulary and the contexts appropriate to it during English class. There are other necessary language components for the students to learn at that time. Therefore, it is necessary for the mathematics teacher to offer the scaffolding students need for access to mathematical concepts. By knowing the language level of each individual student, the teacher can plan appropriate lessons that balance vocabulary development, instruction, modeling, interactive activities, and support.

Types of Language Proficiency

One major concept mathematics teachers need to recognize is the difference between the two types of language proficiency for English language learners. Jim Cummins coined the two types of language Basic Interpersonal Communication Skills (commonly referred to as BICS) and Cognitive Academic Language Proficiency (commonly referred to as CALP) (Crawford, 2004). BICS refers to a student's *social language*. Proficiency in social language requires no specific instruction and typically takes as little as three years to acquire. This knowledge can be acquired through media saturation, music, and social situations. Students can easily seem very capable in social language because they need it to survive. A teacher can often be tricked by a student's level of BICS. The teacher may hear a student chatting with friends and converse with that student before or after class. These conversa-

tions may lead the teacher to believe that the student has a firm grasp of the English language. However, that same student is failing assessments, struggling to keep up with assignments, and unable to write well about mathematical content. This student lacks CALP.

CALP, or *academic language*, takes seven or more years to acquire. CALP is proficiency in the language of the content areas and of the classroom. A student who has strong CALP has command of the use of English within content areas. In mathematics, a student with a strong level of CALP is able to understand key vocabulary, use it in correct context, and write well about his or her understanding of mathematical concepts and procedures. This level of academic language is not learned easily and intuitively, like BICS. This language proficiency only comes with explicit instruction and planned objectives by the content teachers. That is one reason why vocabulary-development lessons are so important for teachers of English language learners to incorporate into mathematics lessons.

Levels of Language Acquisition

Effective mathematics teachers of English language learners also need to know the levels of language acquisition for each English language learner in the classroom. The appropriate lesson for a student who has just moved into the country is going to look very different from the appropriate lesson for an English language learner who is close to being considered fluent in the English language.

Many states have official assessments meant to determine the level at which a student is able to use English. These assessments cover the areas of listening, speaking, reading, and writing. Some of the assessments have a separate score for each domain of language and then a composite score that combines the overall level at which the student is performing in English.

This chart gives information about the levels of language acquisition of English language learners and suggestions to teachers for how to meet those students' needs.

Beginning	Early Intermediate/ Intermediate	Early Advanced/ Advanced
These students fall into a wide range of limited English comprehension. They have minimal or limited comprehension with no verbal production. Some beginning students are just able to give one- or two-word responses. Some are beginning to comprehend highly contextualized information and are able to speak in very simple sentences.	These students have good comprehension of information in context. They may exhibit restricted ability to communicate ideas, but they can usually reproduce familiar phrases in simple sentences. As they improve in proficiency, they improve in the ability to communicate ideas, although they may exhibit errors in production, especially when writing or speaking about highly specialized content.	These students may "trick" teachers into thinking that they are fluent in English. But they often struggle when they have to explain their understanding of an answer or write out the procedures of a concept. They lack the ability to fully communicate higher levels of thinking in content-specific academic language.
Teachers should: • Provide a lot of context for mathematical concepts. • Use physical movement and visuals to explain mathematical vocabulary. • Use sentence frames to help students place mathematical concepts into context. • Ask yes/no questions or questions where the answers are embedded in the questions. • Always include vocabulary-development activities.	**Teachers should:** • Provide visuals and context for mathematical concepts. • Encourage cooperative and interactive activities in order to make mathematical content comprehensible. • Ask questions that require simple sentences with known vocabulary. • Elicit simple explanations and summaries. • Support writing and reading tasks. • Often include vocabulary-development activities and the proper ways to communicate, using the mathematics vocabulary.	**Teachers should:** • Provide structured group discussion of concepts before requiring individual practice and writing about mathematical reasoning. • Elicit explanations that analyze and synthesize mathematical information. • Model the higher levels of thinking with use of specialized vocabulary. • Regularly practice vocabulary-development activities and then take the students to the next levels of higher-level thinking using the vocabulary.

Integrating Vocabulary Development into Instruction

Interactive vocabulary-development activities should be regularly integrated in mathematics lessons in all classrooms. These types of activities are especially necessary for classrooms with English language learners, students struggling with mathematical concepts, or any students who have not shown mastery of the vocabulary.

Teachers should follow these guidelines before beginning to teach the vocabulary activities demonstrated in this resource.

- Decide how long to use one vocabulary activity before introducing a new one.
- Plan for extra teaching time when a new vocabulary activity is being introduced.
- Choose an appropriate activity in order to meet the allotted classroom time for the particular lesson.
- "Frontload" the lesson with vocabulary words before the students need to apply them during practice activities and problems.
- Revisit past vocabulary words in addition to current words, if a lesson requires them.
- Repeat the activity with the same words or new words if it needs to be practiced a few times before the students can correctly perform the activity.
- Clearly state the purpose for an activity, the behavior expectations, and the consequences for not following the expectations if students will be out of their seats.

Vocabulary-Development Activities

These activities often cover the four domains of language—listening, speaking, reading, and writing. It is

important for students to have practice using new vocabulary in a multitude of ways in order for them to take ownership of a word and be able to use it independently. Repetition is as important as exposing students to words in multiple ways.

During these activities, it is imperative that the teacher is actively monitoring the process. The role of the teacher is to enforce procedures (for example, in an oral practice activity, the teacher needs to make sure the students are orally practicing) and to clear up any vocabulary misconceptions or misunderstandings.

These activities can be used as "frontloading" activities, as practice centers for students to use with partners or small groups, or as remedial or enrichment activities during extra instructional time.

Activity 1

Chart and Match

1. The teacher writes the vocabulary words specific to the day's lesson on the board.
2. The students need to have a three-column grid labeled with the following headings: *Word*; *Illustration or Example*; and *Definition or Description*. The students write the vocabulary words down the left side of the grid. One word goes in each row.
3. Next, the teacher introduces the words and leads a whole-class discussion. The teacher uses examples and draws pictures to show the vocabulary words.
4. The teacher then directs the students to draw a picture or to write an example of the vocabulary word in the middle column of the grid, next to the corresponding vocabulary word.

5. In the final column, the class decides on a way to describe or define the word. This should not be a dictionary definition. Rather, after the discussion, the students can write their understandings of the word or the teacher can help them write student-friendly explanations.
6. After reviewing the finished grid, the students cut up the squares. With partners, they can work together to place the three sections for each vocabulary word (the word, the picture or example, and the definition or description) together.
7. The teacher walks around to monitor progress and clear up any misconceptions. With extra time, the teacher can use one grid for a whole-class activity.
8. The teacher hands each student one piece from one cut-up completed grid. If there are not enough pieces for everyone, the teacher can substitute previous vocabulary words or repeat vocabulary words.
9. The students walk around, read their pieces to other students, and trade cards.
10. Then the teacher directs the students to stop, read their final cards, and find the other two students who have their matching components. Each group of three (one student with the word card, one student with the illustration or example card, and one student with the description or definition card) will stand together and present the vocabulary word to the class.

Activity 2

Vocabulary Bingo

In this activity the students create their own boards, which minimizes preparation for teachers.

1. The teacher writes the vocabulary words specific to the day's lesson on the board and gives each student a blank three-by-three grid.
2. The students are directed to write one word in each square. Each student will have his or her words written in different squares, in a different order. With empty squares, the teacher can choose to have the students write previous mathematical vocabulary words, write the vocabulary words twice in two different squares, or allow those empty squares to be "free" spaces.
3. The students are encouraged to draw small pictures or examples next to the location where they placed the word on their bingo charts if there are any words they know.
4. Then the teacher leads a class discussion of the meanings and examples of the day's vocabulary words. The students get more time to write or draw examples to help them understand the vocabulary words they are not familiar with.
5. Next the teacher starts the bingo learning activity. The teacher reads a description of the vocabulary word or shows an example or representative picture.
6. The students locate the word and cover it with a marker. Cut-up pieces of paper or small mathematics manipulatives can be used. If the students wrote the same vocabulary word twice, they can cover both spaces.

7. The teacher should walk around to monitor that students are covering the correct words. Depending on time, the teacher can decide whether students are working to cover a single row, two rows, or "black out" their bingo boards.
8. With extra time, the "winner" can review the words and definitions out loud for the benefit of the whole class. Also, the teacher can have students review the definitions of their covered words with partners.

Activity 3

Which Statement Is Inaccurate?

In this activity, each vocabulary word specific to the day's lesson will be used in four written sentences. The teacher can do this ahead of time, or ask the students to write the sentences once they have been introduced to the vocabulary. For each vocabulary word, three accurate mathematical sentences will be written and one inaccurate mathematics sentence will be written on a transparency. (See the example below.)

1. The teacher displays four sentences on an overhead projector. Three are accurate and one is inaccurate. The sentences are numbered one to four.
2. The students will work in groups of four. The group will read the sentences aloud.
3. Each individual will decide which sentence is inaccurate. They will each hold the number with their fingers while hiding it from their group (e.g., holding three fingers up—but hidden from the group—if number 3 is the inaccurate sentence.)
4. When everyone has decided, each individual will show his or her conclusion to the team. The team will discuss the answer in order to

reach a consensus. When the team has reached a consensus, one student will write down the answer on a small whiteboard or piece of paper.

5. When all teams have reached consensuses, the teacher will ask each team to display its answer to the rest of the class. The class can discuss each team's results.
6. With extra time, the teacher can have the teams convert the inaccurate sentence to an accurate sentence by making appropriate changes.

Sample sentences using the vocabulary word **fraction**:

1. The number $\frac{1}{1}$ is a **fraction**.
2. The number $\frac{3}{4}$ is a **fraction**.
3. The number 65 is a **fraction**.
4. The number $\frac{7}{15}$ is a **fraction**.

Answer: **#3 is the inaccurate statement.**

Activity 4

Sharing Markers

This is a sharing activity. Often, in a group-sharing situation, there are students who love to do all the talking and students who do not participate at all. In this activity, everyone shares vocabulary sentences equally.

1. The teacher shares the vocabulary words specific to the day's lesson. The teacher gives three to four markers to each student. These can be small mathematics manipulatives, small pieces of paper, small candies, or any other small objects. Be sure the teacher checks all the students' medical forms for allergies before bringing food into the classroom.

2. The students form small groups of three to five students.
3. The teacher reviews the rules:
    - Every time a student says a sentence with a vocabulary word in it, he or she gives away a marker.
    - The students have to use all of their markers.
    - Once a student's markers are "spent" he or she is not allowed to say anything more.
    - Students need to be respectful and listen to the vocabulary sentences from all of their classmates.
4. The students generate sentences using the lesson vocabulary words until all students have "spent" their markers.

The sentence can consist of a definition or an example sentence. The teacher may want to share expectations of unacceptable sentences (e.g., I like fractions.).

A student could choose to say all three sentences with three vocabulary words at once. Then that student would be finished speaking and would be directed to listen to the other students in the group. Or, a student could choose to wait until all of the other students are finished before sharing his or her sentences. The teacher should explain that, ideally, the students will take turns, sharing one sentence at a time. This activity ensures that those who need more support can take a little more time to form their sentences. In this activity however, the students should not go around in a circle because English language learners or struggling students may need more time listening before they are ready to share.

Activity 5

Sentence Frames for Vocabulary

This activity will vary depending on the vocabulary being introduced. It is best explained using a specific example, but can be adapted to any set of mathematics vocabulary.

1. First, the teacher shares the vocabulary words specific to the day's lesson.
2. Then, the teacher shares a simple sentence that frames the vocabulary in a proper mathematical context.
3. The sentence frame has blanks in which the students will substitute information. The teacher should write the sentence frame on the board or on a sentence strip. The teacher models complete sentences using the vocabulary.

> This example uses the sample vocabulary words **equation** and **equivalent**.
>
> When a teacher is teaching these words, a common way to express the concept is to say:
>
> The **equation** _____ is **equivalent** to the **equation** _____.

4. The teacher then writes sample answers to put in the blanks.

   For this example, the teacher writes: 3 + 5 = 8; 6 – 2 = 4; 4 + 0 = 4; 9 – 1 = 8.

5. The students work in pairs to practice orally rehearsing the vocabulary with the right substitutions. Younger students would benefit from having actual slips of paper to manually place into the blanks on their sentence strips. Then, they could orally rehearse the information.

> In the example, the students would practice saying:
>
> The **equation** $\underline{3 + 5 = 8}$ is **equivalent** to the **equation** $\underline{9 - 1 = 8}$.
>
> The **equation** $\underline{6 - 2 = 4}$ is **equivalent** to the **equation** $\underline{4 + 0 = 4}$.

6. The teacher directs pairs of students to share the answers they came up with.
7. The partners then work to come up with other substitutions to correctly use with the sentence frame. They practice together and then share with the class.

Post-Reading Reflection

1. Why is it important for students to develop mathematics vocabulary?

2. How are English language learners and below-grade-level students similar in the area of vocabulary development?

3. Identify two ways you can integrate vocabulary development into your instruction.

Building Conceptual Understanding with Manipulatives

Management of Mathematics Manipulatives

Research repeatedly shows that students gain more conceptual understanding and are more successful in demonstrating mastery of concepts when they have had a chance to concretely experience mathematical concepts using manipulatives. In addition, when students use the manipulatives, they perform better academically and have more positive attitudes toward mathematics (Leinenbach and Raymond, 1996). However, many teachers, especially middle school and high school teachers, shy away from using manipulatives.

Often teachers fear that students will misbehave and play with the items, rather than focus on the lesson concepts. Teachers do not always understand how to use manipulatives effectively in order to make abstract concepts concrete and they may dread the extra time necessary to prepare, pass out, and collect the manipulatives. However, as students gain more experience using manipulatives, they will come to appreciate the practice and application that manipulatives add to their mathematical learning experience. Therefore, it is important for teachers to consider the use of manipulatives.

The following section offers general suggestions for managing manipulatives in the classroom.

- First and foremost, **be clear about expectations** while using manipulatives. Make sure that students understand the consequences for misusing the manipulatives (e.g., taking them away).
- **Extra time will be needed the first time** that the class experiences using a new type of manipulative. The teacher should plan for this extra time in order to offer appropriate explanations of how to use and handle the new manipulatives. The teacher also needs to plan extra time for the students to be introduced to the manipulative and to practice using it.
- Before working with manipulatives, give students **time for free exploration** of the materials. One to two minutes is plenty of time. Children's natural inclination is to play and explore. This will give them time to do so before focused work or instruction needs to begin.
- Use **resealable bags or plastic bins** to group and organize the manipulatives together. Label the bags and bins and place them on a shelf. Make

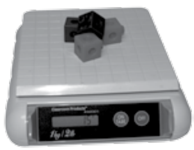
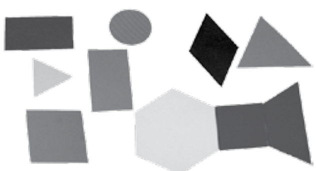
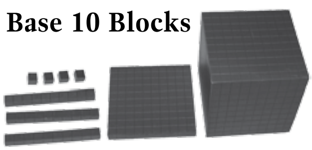
sure frequently used manipulatives are within the reach of the students. This will allow them to gather their materials independently if appropriate.

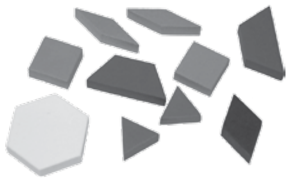
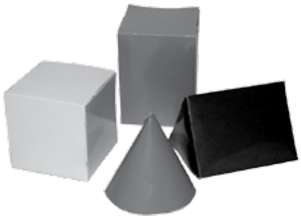
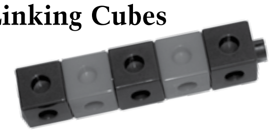
- If manipulatives are commonly used in **small-group settings**, the teacher should **prepare bags of separated manipulatives before the students arrive**.
- If manipulatives are used for **whole-group** instruction, have at least one set of manipulatives for each student or for each pair of students.
- Middle school and high school students should be expected to **clean up and reorganize** the manipulatives in the bags or bins for the next period or the next time they are to be used. The teacher will need to allocate class time for this.
- Create a transparency set of manipulatives for **modeling on the overhead**. If an overhead projector is unavailable, make a large set of the manipulatives and back them with felt, Velcro®, or magnets to display and model with on a board.
- Use **labeled, colored pocket** folders to keep activities with multiple components together. Store folders in cardboard boxes or plastic containers.
- **Laminate the games or instructions on how to use manipulatives** to preserve them for multiple uses.
- **Display charts and instructions on a bulletin board** in the classroom. These charts should be created before the students enter the room for instructional time.
- Occasionally an activity requires the use of many items (manipulatives, paper, markers, pencils, etc.) or the manipulatives are very loud when being used on a table surface. Give the students a small piece of felt to use as a **mat for their manipulatives**. This will reduce the overall classroom noise and enable students to stay organized when working with their materials.

Types of Mathematics Manipulatives and How They Are Used

Manipulatives may vary with the mathematics course, but those in the following chart are commonly used.

Type of Manipulative	Possible Uses in a Mathematics Classroom
Rulers metric and standard 	• measurement • length • standard and nonstandard units • selecting appropriate tools of measurement • comparing metric and standard measurements
Calculators 	• calculations • number and operations concepts • algebra equations • problem solving • math facts • addition, subtraction, division, and multiplication • area
Computers 	• number and operations concepts • equations spreadsheets • graphing • problem solving • algebra equations
Geoboards 	• geometry • sorting and classifying • describing • drawing • symmetry • spatial reasoning

Type of Manipulative	Possible Uses in a Mathematics Classroom
Number Lines 	• number and operations concepts • positive and negative numbers • fractions • ordinal and cardinal numbers • addition and subtraction
Number Cards 	• numbers and operations concepts • positive and negative numbers • ordinal and cardinal numbers • addition, subtraction, division, and multiplication
Dice 	• application games • math-facts games • addition, subtraction, division, multiplication • number sense • prediction and statistics
Scales metric and standard 	• measurement • standard and nonstandard units • weight • selecting appropriate tools of measurement • comparing metric and standard measurements
Pattern Blocks 	• number sense • whole numbers • algebra • fractions • spatial visualization/estimation • patterns • representation of data and concrete objects
Base 10 Blocks 	• number sense • place value • addition, subtraction, division, and multiplication • area and volume

Type of Manipulative	Possible Uses in a Mathematics Classroom
2-D Shapes 	• geometry • sorting and classifying • comparing and ordering • recognizing shapes • describing • drawing • area and perimeter
3-D Shapes 	• geometry • sorting and classifying • comparing and ordering • recognizing shapes • describing • drawing • area, perimeter, volume
Play Money 	• equations with money • real-life situational word problems • problem solving
Flash Cards 	• numbers and operations concepts • math facts • addition, subtraction, division, and multiplication
Linking Cubes 	• number sense • addition, subtraction, division, and multiplication • comparing and ordering • area and volume
Fraction Bars 	• comparing and ordering • addition, subtraction, division, and multiplication • number sense • finding equivalent parts • fractions

Preventing Manipulative Dependency

Another common fear among teachers using manipulatives is that students will become dependent on their use. Students need to have fluent procedural abilities. They need to compute and calculate equations correctly and use formulas and mathematical rules swiftly. However, this process usually comes more readily when students are exposed to multiple representations of concepts by first exploring them concretely and then learning how to proceed using the experiences they gained from using manipulatives.

Manipulatives are meant to help students use concrete objects to correspond to mathematical ideas. Teachers should devise lessons in which students learn to organize their thinking in concrete ways. However, teachers also need to move students past the concrete into pictorial and abstract thinking.

Manipulatives can be a great tool for engaging students in learning new concepts. They are especially helpful when teachers are trying to reach kinesthetic, visual, and English language learners.

Simply placing the manipulatives in front of students is not enough to guarantee that they will learn from them. Always providing them, without instructing students in how to begin to solve the same problems without them, will also prove detrimental to students when they are later assessed without having access to the manipulatives. Teachers need to work with students over time to help them connect the object, the symbol, and the mathematical concept that is represented.

The information on the next page highlights the progression teachers need to follow in order to prevent manipulative dependency.

1. **Introduce** manipulatives. **Explain** the role of manipulatives, how they connect to an overall mathematical concept, and the expectations for student use.
2. Give **students practice** in using the manipulatives to explore the mathematical concept.
3. **Model** the mathematical concept with **pictures** that replace the manipulatives. Make connections between the manipulatives and the pictures.
4. Give students **practice** in using **pictures** (as a substitution for the manipulatives) to explore the mathematical concept.
5. Teach the **abstract qualities of the mathematical concept**. Make connections between the pictures and the equations or formulas.
6. Provide ample opportunities to **practice** solving the equations **without pictures or manipulatives**.
7. Return to manipulative use when needed, but **repeat the entire process** to move students to abstract thinking and problem solving.

Post-Reading Reflection

1. What was your view of manipulative use in the classroom prior to reading this chapter? If it changed after reading this chapter, explain how.

2. Choose a manipulative you have access to in your classroom. Write a word problem that involves using that manipulative to solve the problem.

Teaching the Procedure

Building on Understanding

By its very nature, mathematics instruction implies learning skills that must build on one another (Dean and Florian, 2001). Students must use what they already know in order to apply it to any new learning. Therefore, it is vital that teachers consider the *process* of building on understanding as they approach their mathematics curriculum each year.

1. **Start with the mathematics content standards, not the textbook.**
   - The next grade level's teacher will expect students to come in with these skills covered in the content standards. These concepts are assessed at the end of the school year.

- A teacher cannot open a mathematics textbook and teach cover-to-cover because textbooks may cover nonrequired concepts and often give only a little concept introduction, which might not be adequate for struggling students.
- The content standards often introduce a concept in one grade and then revisit it at a more in-depth level the following year(s).

2. **Find out what they already know.**
    - Teachers should look over the concepts that were supposed to be covered in previous years, as well as what students will be required to master in the following year.
    - Teachers must not assume that the students have mastery in what they were supposed to cover during the previous year.
    - Before introducing a concept, assess prior knowledge (Do they know what they are supposed to know from last year? At what level will you be able to begin the lesson?), and give the necessary background knowledge (Is there something they need to know before they can start learning about this concept?).
3. **Build understanding for every new procedure being taught.**
    - Directly teach and model the new concept. Give many examples that build on the students' prior knowledge.
    - Post and explain the procedure. Draw pictures, when helpful.
    - Give students sufficient guided practice.
    - Allow the students to verbalize procedures.

- Monitor their progress. Ask questions and allow them to ask questions so they can clarify understanding and build off what they already know.
- Allow students to practice with and explain concepts to peers.
- Make sure that the independent practice matches what they were practicing in guided practice, rather than giving them completely different or harder problems to solve once they are removed from any support.
- Review the procedure before moving on to the next aspect of that mathematical concept or before adding a more challenging level.

Teaching More Than One Way

Students need to learn a lot in a single academic year and sometimes they don't learn it all the first time. When older students fail a mathematics course, they are often required to repeat it, facing the same textbooks, teaching strategies, routines, curriculum materials, and sometimes even the same teachers. When younger students fail to grasp a mathematical concept, they are often given extra practice work that presents the material in the same way they did not understand in the first place.

Students who experience "the same—but a second time" are basically being set up to fail once again. It is a detrimental cycle that denies students the right kind of intervention and differentiation to meet their needs.

1. Consider the **multiple intelligences** of the students, introduced by Howard Gardner in *Frames of Mind* (1993). When teachers allow students to use their own strengths toward learning about mathematics, students will have more success. Beyond the mathematic/logical intelligence, teachers can explore more linguistic, kinesthetic, musical, or social ways to investigate mathematical concepts.
2. Consider the students' **learning styles**. If only oral instructions are given, then the visual learners might be struggling. Offer auditory, visual, and kinesthetic ways of direct procedure and guided practice.
3. **Change your teaching techniques**. If you usually use overheads, try having students create posters. Use cooperative groups in which students first solve problems individually and then show their answers to the other three group members, come to a group consensus, and finally present the agreed-upon solutions to the rest of the class. Try different manipulatives than you used the first time.
4. **Experiment with partner activities**. Alternate among peer practice, teacher-directed partner activities, guided concept explanation, peer tutoring, finding multiple partners to share concepts with, and random mixing.
5. Remember that learning a new mathematical concept is a process. Scaffold easy to hard, and strong teacher support to independent work. **Start with the concrete, move to the abstract, and finally move to the application of the mathematical concept.**

Taking Notes

Taking effective notes is not a skill that students learn independently. Rather, they need to be **explicitly taught how to take notes in an organized manner**. Good mathematics notes are going to look very different from good notes in another subject. Therefore, it is the responsibility of mathematics teachers to teach their students how to best organize mathematical information.

- Math journals usually have an element of student freedom and choice. Students can copy examples of mathematics problems, complete independent practice problems, write down their understanding of procedures, and note questions to ask the teacher. Teachers can periodically collect math journals and quickly check students' progress.
- Math learning logs are teacher-directed notes that students can later refer to when they are working independently or studying for upcoming tests. Usually, there is a set format that students use for each new major mathematical concept. Teachers should hold students accountable for the neat organization and complete inclusion of vital information. Teachers should collect learning logs often to check for accuracy of recorded information, neat organization, and inclusion of the central information. Before a test, teachers should give students practice in class at reading notes and using them to rehearse and remember key information that they will need to know. They can even quiz partners about information in the notes.

Consider the following skills that are involved in well-written **math learning log notes**.

- The crucial vocabulary needs to be identified and defined.
- The procedure should be written correctly and concisely.
- The practice problems should be kept separate from the organized key-idea notes.
- Some mathematical concepts are best described with illustrations.
- Students should be taught how to identify the central idea in each page of notes.
- There should be at least one correct example of how to solve a problem.
- No extraneous, unhelpful information should be included.
- The notes should be legible and well organized.
- Color coding or highlighting skills should be taught to students and applied to good note taking.

Creative Ways to Practice

Students need ample opportunities to think about mathematical concepts, relate them to their everyday lives, and successfully perform mathematical functions.

Teachers get frustrated when they spend a whole period teaching a concept and then get back poorly completed homework. Often, teachers are spending too much time talking and not allowing enough time for effective student practice. Effective practice does not mean students complete the even numbered problems in class and the odd numbered problems for homework. This strategy does not allow students to receive adequate practice for successful application and mastery of mathematics.

The most important thing teachers need to do is to **allot sufficient time for practice**. Teachers need to **structure practice time** with guided-practice activities so that students are held accountable for practicing.

The following creative activities allow students interesting ways to practice mathematical concepts.

Peer Explanation Activity

After students have been introduced to a new mathematical concept, the teacher can pair them with partners and give each pair a different problem to solve. Then, the teacher can direct each pair to join another pair and explain their solution to one another. This activity can help them reinforce procedure and concept understanding.

Whiteboard Cooperative Group Activity

Students work in groups of four, each with a number from one to four. Each group has one small whiteboard and one pen. The teacher displays a problem and directs all the students to solve the problem individually on paper. Then, the group members compare their solutions. If someone has a different answer, the group can discuss the procedure. Once everyone has the correct solution, the teacher chooses a number from one to four. The group member whose number corresponds to the number given by the teacher records the group solution on the small whiteboard. That person displays the answer to the class before receiving a new problem to solve in the same way. If any group has a different answer, the class discusses the solution together with the teacher to decide which answer is correct.

Oral Rehearsal Activity

The "language of mathematics" requires very specific formation of English syntax and new terms. Therefore students need chances to practice "speaking mathematics." In the typical question/answer classroom dynamic, only the confident students raise their hands to answer questions. Teachers need to structure oral practice so that *every* student is thinking of answers and being allowed to share them with a partner. In another oral activity, the teacher can display a problem on the board, ask for a volunteer to read it, and then continue with that *same* example (rather than writing out another one) so that *each* student can choose to read it. An example would be reading a place value number, such as 253,467 correctly. Thus, each student gets the opportunity to orally practice, and all students can participate within their own comfort levels.

Graphic Organizer Review Activity

Before a test or at the end of a unit, the teacher divides the class into pairs. The teacher then gives each pair of students matching colored paper. No two pairs can receive the same color. Each pair graphically displays the important information from that unit on its papers as a review or study guide. The pair should decide on a way to display the information that makes sense to each of them and clearly shows the information the pair wants to include. The pairs should be encouraged to write explanations or steps, draw pictures or diagrams, and show sample problems that help them review any mathematical concepts and problems they have been learning. Each member of a pair should have the same information on his or her paper.

When all the pairs are finished creating their graphic organizers, the students split from their pairs to find new partners with different colored papers. They each

share their graphic organizers and verbally explain the information they included. Next they add any new information learned from the other students that was not already included on their papers. Then they each find new partners and repeat the same sharing process. The teacher can decide to set a time limit on this activity or require each student to share with a set number of other students. Using this activity allows students multiple opportunities to review and reinforce crucial information.

Working as a Team Cooperative Activity

Students can work in pairs or in small groups. Each person on the team needs a different colored pencil. The teacher displays a problem. The students rotate, each writing one step and passing the paper. While the other students may offer suggestions, help, or advice, they may not take the colored pencil from the student whose turn it is. The students continue to rotate until the problem is solved. As the teacher monitors the work, he or she can quickly assess who contributed which portion of the problem solution by looking at the colors.

Post-Reading Reflection

1. What aspects of instruction are important in teaching the procedure?

2. Describe two new ways to present information that you learned from this chapter and why it is important to do so.

3. Think back to how or if you were ever taught to take notes. What strategies would have been helpful to you when you were in school? Why is note taking a valuable skill to teach your students?

Teaching Problem Solving

Why Teach Problem Solving?

Today's world is changing rapidly. Many of the changes mean that proficiency in basic mathematical concepts will become more and more critical.

Furthermore, mathematical reasoning and problem solving will be crucial to the success of today's students as they work to find solutions to problems in everyday life. Just knowing the basic facts and formulas is not enough to solve the wide range of problems and situations that arise in life. It is no surprise, then, that problem solving is an important, current mathematics focus in the classroom.

While students must be skilled at performing computations required to find mathematical solutions, this is only part of the process. Before students begin to manipulate the information in a problem, they should understand its meaning and plan a way to solve it.

Students, therefore, need to learn about problem solving as a process and the strategies they can apply to find solutions (Kilpatrick, et al., 2001). The process of problem solving goes beyond finding simple solutions; it encourages students to reflect on the solutions, make generalizations, and extend problems to include new possibilities for investigation. Once students learn the process of problem solving, they can use mathematical approaches to solve real-life problems.

The pages that follow provide explanations and examples of 12 problem-solving strategies that can be adapted to meet students' needs. Information about each strategy will provide insight into ways a particular strategy can be used in the classroom. Examples are given for each strategy. They demonstrate the application of the strategy to the solution of the problem. These examples are not appropriate across all grade levels. They are only used to demonstrate the use of the strategy.

Steps for the Problem-Solving Process

It is important that students follow a logical and systematic approach to their problem solving. These four steps will enable students to tackle problems in a structured and meaningful way. These steps are not intuitive for learners. Therefore, teachers will need to plan instructional time to explicitly teach the process, model it, and finally allow for ample opportunities for guided and individual practice as students approach the problem-solving strategies.

Step One

Understanding the Problem

Encourage students to read the problem carefully a number of times until they fully understand what it asks. As students are learning this step and progressing toward internalizing it, the teacher will allow time for students to discuss the problem with peers or rewrite the problem in their own words. Students should ask internal questions such as, "What is the problem asking me to do?" and "What information is relevant and necessary for solving the problem?" (This will need to be repeatedly modeled for the students in the learning process.)

Next, students should underline any unfamiliar words and find their meanings. Selecting the information they know and deciding what is unknown will help them begin too see how to solve the problem. They should also see if there is any unnecessary information. It will be helpful for teachers to model these processes until students understand how to complete them on their own.

Step Two

Planning and Communicating a Solution

Students should decide how they would solve the problem by thinking about the different strategies that could be used. Sometimes it will be necessary for students to use more than one strategy to solve a problem

They could try to make predictions, or guesses, about the problem. Often these guesses result in generalizations, which help to solve problems. Students should be discouraged from making wild guesses, but they should be encouraged to take risks. They should always think in terms of how this problem relates to other problems that they have solved.

As they attempt different strategies, they should keep a record of those they have tried so that they do not repeat them.

The 12 strategies in this book include:

- drawing a diagram
- drawing a table
- acting it out or using concrete materials
- guessing and checking
- creating an organized list
- looking for a pattern
- creating a tree diagram
- working backwards
- using simpler numbers
- open-ended problem solving
- analyzing and investigating
- using logical reasoning

Other strategies include:

- breaking down ideas into smaller pieces
- writing a number sentence
- writing down ideas as work progresses so students do not forget how the problem was approached
- approaching the problem systematically
- rereading the problem in order to rethink strategies if the student becomes "stuck"
- orally demonstrating and explaining how an answer was reached

Step Three

Reflecting and Generalizing

Many times the solution and strategies in one problem can help students know how to solve another problem.

Therefore students need to learn the importance of reflecting on the work they have done. Teachers need to teach students the critical process of reflection. This process should be modeled as teachers show problem solutions. Teachers can even solve problems incorrectly in order to go through the reflection process and "catch" mistakes. Students need to decide if their answers make sense and if they have answered the question that was asked. They should illustrate and write their thinking processes, estimations, and approaches. This gives them time to reflect on their practices and grow in the use of problem-solving strategies. When they have an answer, they should explain the process to someone else.

Step Four

Extension

This step also needs to be explicitly taught and modeled by teachers because students need practice in internalizing it. Students need to learn how to ask themselves "what if" questions to link this problem to another. This will take their explorations to deeper levels and encourage their use of logical thought processes. Students should also decide if it is possible to solve the problem in a simpler way.

Problem-Solving Strategies

Strategy One—Drawing a Diagram

This strategy often reveals aspects of the problem that may not be apparent at first. A diagram that uses simple symbols or pictures may enable students to see the situation more easily and can help them keep track of the stages of a problem in which there are many steps. Students need to develop the skills and understanding to use diagrams effectively.

- Using a simple line drawing to symbolize an object will help a student visualize a situation.

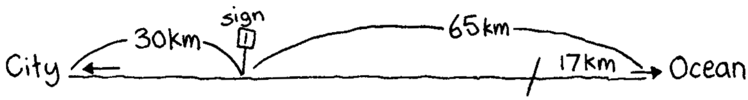
Example: How many markers would be needed if you placed a marker at every two-meter point on a ten-meter rope?

In response, students may calculate mentally 10 ÷ 2 = 5, so five posts are needed. However, if students draw the rope and markers, they will see that six markers are needed because an additional one is needed for the starting point.

- Using a time/distance line to display the information helps to show distance or movement from one point to another.

Example: A signpost is placed on the highway. It says that the city is 30 kilometers to the west and the ocean is 65 kilometers to the east. How far are you from the city when you are 17 kilometers from the ocean?

Students should draw a line and write the distances on it.



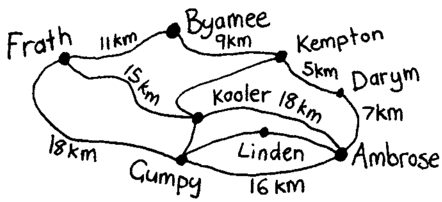
30 km + (65 km – 17 km) = 78 km

- Students will need to scale down diagrams if large areas need to be drawn. Show students how to use scaled-down measurements to solve a problem and then convert the solution to the actual measurements.

Example: In a drawing, one centimeter could have the value of one kilometer. Alternatively, a one-centimeter line could represent 10 kilometers or even 500 kilometers, depending on the scale of the drawing.

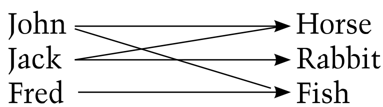
- Students may need to map or show directions when drawing diagrams, or use maps as the focus of a problem.
- They need to understand how to plot a course by moving up, down, right, or left on a grid, or use the compass points to direct themselves—north, south, east, west, northeast, southwest, and so on.

Example: Plot four different routes from Byamee to Gumpy without passing through any town twice per route.



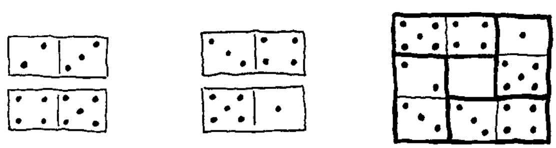
- Students will find it helpful to draw diagrams and use symbols to show the relationships among things.

Example: John, Jack, and Fred love animals. John's favorite animals are fish and horses. Jack's favorite animals are horses and rabbits. Fred's favorite animals are fish. Which boys have fish as their favorite animals?



- Drawing pictures can help students organize their thoughts and simplify a problem.

Example: Organize the four domino pieces in a square shape with each side of the square adding up to a total of 10.



Strategy Two—Drawing a Table

This strategy helps students to organize information so that it can be easily understood and relationships between one set of numbers and another become clear. A table makes it easy to see the known and unknown information. The information often shows a pattern or part of a solution, which can then be completed. It also can help reduce the possibility of mistakes or repetitions. Students need to develop the skills and understanding to create and use tables effectively.

- Teachers may have to help students decide on the number of columns or rows to fit the variables.
- First decide how to classify and divide the information. Establish the number of variables to be included in the table. Then decide on the number of rows, columns, and headings.

Example: There are 18 animals at the farm. Some are chickens and others are cows. There are 70 legs that are visible. How many of each type of animals can be seen?

Students will need to draw a table that has three columns.

Number of Chickens	Number of Cows	Number of Legs

- Often a pattern becomes obvious when creating a table. Students may leave gaps in the tables and complete the patterns mentally or follow the pattern in the table to find the information the problem is asking for.

Example: Two people are being compared in this problem: Mrs. Crawford is 32 years old and her daughter Lisa is eight years old. How old will Lisa be when she is half as old as her mother?

A two-column table is drawn. By leaving gaps and calculating mentally, we established that when Lisa is 24 years old, her mother will be 48 years old.

Lisa	**Mrs. Crawford**
8	32
9	33
10	34
11	35
12	36
13	37
24	48

This second example shows how a pattern can be established.

Example: A child is playing a game of basketball by himself in the park. Then, at regular intervals, other groups of students begin to arrive at the park. From each new group, two children decide to join the basketball game. The first group has three children, the second group has five children, and the third group has seven children. How many groups will have appeared by the time there are 64 people in the park?

Three columns are needed for the table. The columns should be headed *Groups*, *People*, and *Total*.

Groups	People	Total
	1	1
1	3	4
2	5	9
3	7	16
4	9	25
5	11	36
6	13	49
7	15	64

Seven groups will have appeared.

Strategy Three—Acting it Out or Using Concrete Materials

This strategy uses objects or materials to represent people or things in the problem. This helps students visualize the problem in a concrete way and find the solution.

- A variety of objects such as beans, counters, blocks, toys, or erasers can be used to symbolize people or places. These objects can be moved through the steps of the problem. It is important to chart this movement to keep track of the process.
- Students can also act out the roles of the different participants depending on the situation in the problem.

Example: Their grandparents sent Leslie, Randall, and Carla $190, in total, for their birthdays. Their parents had to divide the money up so that Randall was given $20 more than Leslie, and $30 more than Carla. How much money were they each given?

Ask three students to act out the parts of the children and use \$190 play money for the exercise.

Start by giving Leslie an estimated amount.

Give Leslie \$50.

Randall should be given \$20 more: \$50 + \$20 = \$70

Carla should be given \$30 less than Randall:
\$70 – \$30 = \$40

Total \$50 + \$70 + \$40 = \$160. This total is too low.

Start with giving a higher amount.

Give Leslie \$60.

Randall should be given \$20 more: \$60 + \$20 = \$80

Carla should be given \$30 less than Randall:
\$80 – \$30 = \$50

Total \$60 + \$80 + \$50 = \$190. This is correct.

Strategy Four—Guessing and Checking

This strategy allows a student to make an educated guess and check the guess against the problem. If it is not a correct solution, the student revises the guess and checks until a correct solution is found.

- Each student begins by finding the important facts in the problem and making a reasonable guess based on the information. Teachers will want to help students when they first start using this strategy. Students will learn from their faulty guesses.
- Students then check their guesses against the problem and revise their guesses until they find the correct answer. Creating a table is a good way to make sure all guesses are checked.

Example: Alana is five years older than Saul. Alana's age plus Saul's age totals 25. What are their ages?

Students should first note the important information given to them in the setup of the problem. In this problem, the important information is that:

Alana is five years older than Saul.

Alana's age plus Saul's age totals 25.

Students should create tables to help them solve the problem.

Guess	Alana's age	Saul's age	Total

Students should now test their guesses.

Guess	Alana's age	Saul's age	Total
1	12	7	19

Too Low

Guess	Alana's age	Saul's age	Total
1	12	7	19
2	18	13	31

Too High

Guess	Alana's age	Saul's age	Total
1	12	7	19
2	18	13	31
3	15	10	25

Correct

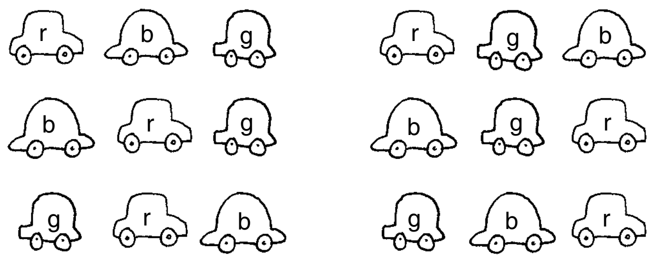
Strategy Five—Creating an Organized List

This strategy is used instead of a table when a greater amount of information is available. Students need to follow a procedure or sequence to find the solution to the problem and make sure no information is left out or repeated.

- When creating the list, one item should remain constant while the others change. Students need to write down the processes they are using to stay on track.
- Students can work sequentially, using the information from the problem or filling in the gaps of a pattern once it is created. Systematic work is key to being successful in this strategy.

Example: Shaun has three toy cars that he keeps on his bookcase. One is red, one is blue, and the third is green. He likes to change the order in which he displays them. How many different ways can he do this?

To solve this problem, place the red car (r) first, and then place the other cars in their two possible positions.



There are six different possibilities.

Strategy Six—Looking for a Pattern

This strategy is an extension of drawing a table and creating an organized list. It is often used because mathematical patterns can be found everywhere.

There are many ways to check for a pattern.

- Find the difference between two consecutive numbers.
- Find out if the numbers are rising or falling in a regular sequence.
- Decide whether the numbers have been multiplied or divided by any given number.
- Once a pattern is found, it can be extended or continued.

Some patterns include two operations.

Example: 6 9 8 11 10 __ __

This pattern includes two operations: + 3, – 1

Example: When Jacklyn went strawberry picking, one out of every six strawberries had wormholes. How many good strawberries were there out of 84?

Look at this table. We can see that a pattern has been established. The "Good" column is increasing in multiples of 5, and the "Bad" column is increasing by 1s. So if there is a total of 84 strawberries picked, 70 will be good and 14 will be bad.

Good	Bad	Number of strawberries
5	1	6
10	2	12
15	3	18
20	4	24
25	5	30
30	6	36
35	7	42

Strategy Seven—Creating a Tree Diagram

This strategy uses a diagram with different branches to represent relationships between different factors in a problem. A tree diagram enables the students to visualize the different factors in the problem and ensures that no factors are repeated or missed.

- First the students need to identify the important factors in the problem and list them. Then the factors need to be connected using lines or parentheses.
- When the problem is complete, it should be checked to make sure that all factors are properly connected and no information has been left out.

Example: A class of students was asked to list its favorite fruits. In the list the students included apples, oranges, and bananas. Once the initial list had been compiled, students were asked to organize the three types of fruit in order, from their least favorite to their most favorite. The students in the class covered every possible combination. How many combinations were there?

Most favorite ⟶ Least favorite

Apples
- oranges — bananas
- bananas — oranges

Oranges
- apples — bananas
- bananas — apples

Bananas
- oranges — apples
- apples — oranges

Strategy Eight—Working Backwards

This strategy is used to solve problems that include a number of linked factors or events, where some of the information has not been provided. The object is to determine the unknown information. The events occur one after the other and each stage or piece of information is affected by what comes next.

- To solve the problem, begin with the ending information and work backwards until the problem is solved. It is important to remember that mathematical operations will have to be reversed.

Example: Arnold baked cupcakes over the weekend. Each day during the week he took three cupcakes to school to share with his friends. On Saturday, when he counted, there were 18 left. How many cupcakes had he baked?

Begin with the information you know—the number of cupcakes Arnold ended with—and work backwards.

Arnold has 18 cupcakes at the end of the week.

On Friday he brought three cupcakes to school: 18 + 3 = 21

On Thursday he brought three cupcakes to school: 21 + 3 = 24

On Wednesday he brought three cupcakes to school: 24 + 3 = 27

On Tuesday he brought three cupcakes to school: 27 + 3 = 30

On Monday he brought three cupcakes to school: 30 + 3 = 33

Arnold made 33 cupcakes over the weekend.

Strategy Nine—Using Simpler Numbers

This strategy can be used to solve a difficult or complicated problem in order to simplify the numbers.

- Begin by substituting smaller numbers for larger numbers to make the calculations easier. Then use the same steps to solve the original problem.
- Or solve a series of simpler problems to see if a pattern emerges and apply the pattern to the more complicated problems.

Example: It took 16 artists 10 hours to paint half a mural. Only 4 of the artists can stay to finish the other half. How long will each artist work if there are only 4 artists to complete the other half?

Start with a simpler example.

If it takes 4 artists 8 hours to paint half a mural, how long will it take 2 artists to paint the other half?

First, find out how long it would take 1 artist to paint half the mural all by himself.

He would have to work four times longer to do the job of 4 artists, so he would take 32 hours.

4 x 8 = **32**

If 2 artists work on the other half, each will only have to work half as much time as one artist, so it would take them only 16 hours.

32 ÷ 2 = **16**

Now solve the original problem.

We know that 16 artists take 10 hours, so 1 artist would have to work 16 times longer to do the job of 16 artists, so he would take 160 hours.

16 x 10 = **160 hours**

If 4 artists work on the other half, they would each have to work only one fourth as much time, so they would each work 40 hours.

$$160 \div 4 = \mathbf{40}$$

Strategy Ten—Open-Ended Problem Solving

This strategy is used to explore problems that might be answered in a number of ways. Although finding the correct answer is important, teachers should value the student's process for solving the problem and gain insight into the student's developmental understanding.

- When structuring problems, words such as *create*, *make*, *design*, *explore*, and *investigate* should be used.
- Students should use processes such as labeling counters to visualize the problem, trying different combinations of numbers, and working to find as many solutions as possible.

Example: Investigate which combinations of the digits 3, 4, 6, and 7 will create addition problems that have a sum falling between 100 and 120.

$$\begin{array}{r} 43 \\ +\,76 \\ \hline 119 \end{array} \quad \begin{array}{r} 34 \\ +\,67 \\ \hline 101 \end{array} \quad \begin{array}{r} 43 \\ +\,67 \\ \hline 110 \end{array} \quad \begin{array}{r} 34 \\ +\,76 \\ \hline 110 \end{array}$$

$$\begin{array}{r} 37 \\ +\,64 \\ \hline 101 \end{array} \quad \begin{array}{r} 36 \\ +\,74 \\ \hline 110 \end{array}$$

Strategy Eleven—Analyzing and Investigating

When using this strategy, analyze what is known and what needs to be known. Use the known information to investigate the problem and collect data.

- Begin by making an estimate. Estimating is an effective method for gauging the reasonableness of an answer.
- After estimating, plan an approach to solve the problem. Determine what is involved in the task and what strategies should be used to gather information. Decide how the information gathered should be presented.

Example: Each weekday, cars continually stream past the school. How many weekdays will it take for a million cars to pass the school?

One possible solution

Students believe there are two peak traffic times, 8:00–9:00 A.M. and 3:00–4:00 P.M. During the remainder of the day, cars do not pass often.

Plan: Survey the traffic during the peak hours and at the other times during the day. Tally the number of cars that pass during a 10-minute period, and multiply by six to get the number of cars per hour.

Peak Hour

8:00–9:00 A.M. 250 cars in a 10-minute period,
250 x 6 = 1,500 cars/hour.

3:00–4:00 P.M. Assume it is the same, 1,500 cars/hour.

Other Times

1:00–2:00 P.M. 10 cars in a 10-minute period,
10 x 6 = 60 cars/hour.

The period between midnight and 6:00 A.M. would be very quiet—estimate 5 cars per hour during this time.

The flow of traffic in one 24-hour period can be recorded as follows:

Peak Hours = 3,000 cars

Midnight to 6:00 A.M. = 5 cars/hour x 6 hours = 30 cars

All Other Times = 60 cars/hour x 16 hours = 960 cars

Total Cars in One weekday = 3,000 + 30 + 960 = 3,990

1,000,000 divided by 3,990 cars/day = 250.6 weekdays until one million cars pass the school

Strategy Twelve—Using Logical Reasoning

This strategy is used when the problem gives information as pieces of a puzzle. Each piece of information is important to solve the problem. Process of elimination is one approach in this strategy, where each piece of information builds to the solution. Drawing a grid to organize given information is also an approach that can be used.

- Students start by reading each clue carefully and thoroughly. Often the clues need to be dealt with in an order different from how they were presented. Then the students take the steps necessary to solve the problem.

Example: Julie, Yushiko, and Sam are each about to eat a sandwich for lunch. On the plate there is a tomato sandwich, a honey sandwich, and a peanut butter sandwich. Use the following clues to work out which sandwich belongs to each person.

Julie's sandwich has salt and pepper on it.

Sam is allergic to nuts.

Yushiko dislikes sweet things.

By drawing a grid, students can use the information they learned in the problem to cross out information and visualize the correct answers.

Sam cannot have the peanut butter sandwich if he is allergic to nuts. Yushiko will not eat the honey sandwich because she does not like sweet things. Julie's sandwich has salt and pepper on it, which means it is not the peanut butter or the honey sandwich. From this information you can fill in the grid to show who gets which sandwich.

	tomato	honey	peanut butter
Julie	✔	✘	✘
Yushiko	✘	✘	✔
Sam	✘	✔	✘

How to Create Word Problems

When teachers are creating word problems to use with their students, it is important that they keep several things in mind.

- Decide the problem-solving strategy being utilized before writing the problem.
- Use appropriate vocabulary for the students in the classroom.
- Keep the sentences simple.
- Only include as many problems as the students can complete in one sitting.

- Work the problem at least twice before giving it to the students.
- Have another teacher work the problem to check for clarity, vocabulary/word choice, and correct mathematics.

Here are some common words found in word problems for teachers to use when they are creating word problems. These words can also be used with students to give them clues for which operation to use when reading a word problem.

Adding	Subtracting	Multiplying	Dividing
increased	decreased	of	into
more than	smaller	times	split
combined	reduce	multiplied	half
together	minus	per	average
total	less	each	cut
sum	difference between/of	product of	per, a
added to	less than	factors	out of
plus	fewer than	twice (2)	ratio of
additional	loss	double (2)	quotient of
together	take away	triple (3)	percent
joined	left	fraction of	divided by
altogether	remains	at this rate	divided equally
gain			each
raise			fair share
both			
in all			

Post-Reading Reflection

1. Choose a problem-solving strategy to use with this problem. If more than one strategy can be used, make notes about how you could help your students determine which strategy is most efficient to use. A doll's dress can be created with $\frac{4}{5}$ of a yard of material. How many dresses can be created if you have 22 yards of material?

2. Choose one problem-solving strategy. Write a word problem that utilizes that strategy to solve the problem.

Integrating Mathematics Across the Curriculum

Teachers of mathematics are bound one day to be asked the age-old question, "But when am I *ever* going to use this?" By integrating mathematics across the content areas, students will realize the importance of mathematics in their daily lives and how often it is used in subjects other than mathematics.

Integrating Mathematics and Literacy

Mathematics is often inaccurately perceived as "strictly numbers." While it is true that mathematics is heavy in numbers, orders of operations, and procedural formulas, there are also plenty of opportunities to examine the possibilities for connecting reading and writing to key mathematical concepts.

The Language of Mathematics

There is actually a language of mathematics that students have to learn to read, interpret, speak, and write as they tackle mathematical concepts. The linguistic element of mathematics varies by culture, as well. For example, the way fractions are read in English is different syntactically from reading fractions in other languages. English starts with the numerator (a cardinal number) and then the denominator (an ordinal number.) There are exceptions: students do not say $\frac{1}{2}$ as "one second," but rather "one half." In Japanese, to express a fraction, the student first states the denominator and then the numerator. "One fourth" literally translated from Japanese is "four sections of which we are referring to one section."

Reading in a Mathematics Class

Content areas such as mathematics lend themselves to thematic modes of instruction. Within these themes, students are given a cognitive net of vocabulary, background, and concepts with which to connect the new learning and understanding (Moyer, 2000). This is especially important for English language learners who are learning language and content concepts simultaneously. A student must be able to read concept introductions, explanations, word problems, examples, and instructions in a mathematics book or within an activity. Additionally, students need to learn certain terms and phrases to understand what formulas, procedures, and orders of operation to apply to word problems. These are specific to the mathematical content area. A language arts teacher will not teach students how to navigate and comprehend the reading that is necessary in a mathematics textbook. It is up to the mathematics teacher to model and explicitly demonstrate the necessary reading skills for the successful reading that is necessary in mathematics.

How to Integrate Fiction and Nonfiction

A teacher may apply books to lessons to help contextualize mathematical concepts. Some of the skills that students need for reading are also necessary in mathematical thinking. These include summarizing, sequencing, and finding the main idea. Integrating reading and mathematics can improve students' general language skills and their abilities to communicate and express themselves mathematically (Moyer, 2000).

Many students enjoy reading and listening to stories because fiction text is appealing and interesting. There are many fiction texts available that directly explain mathematical concepts and pose specific mathematical problems to solve in a fun and exciting way. Additionally, fiction books often present opportunities to teach logic through interesting problems that are established by the characters. Fiction books often present mathematics in an unintimidating way for students to understand and can be used with students of all ages.

Reading comprehension skills are essential for academic success throughout all grade levels. Students are often taught strategies for comprehending fiction texts. However, "[i]t's essential that students learn from the earliest grades through high school how to read nonfiction, if they are to survive and thrive in an adult world crammed with information" (Collier, 2006). Nonfiction texts naturally provide real-life situations and information that students can use to understand and apply mathematical concepts as well as reading comprehension strategies.

Nonfiction and fiction texts can be used for discussions about the following mathematics concepts:

- counting/general number sense
- estimation
- addition, subtraction, division, and multiplication

- fractions, decimals, and percentages
- measurement
- simple algebraic functions and inequalities
- logic and probability
- researching, collecting, and comparing information and data to analyze and display in graphs

Writing

As students discuss and read about mathematical concepts, teachers can also guide them in their development of writing the language of mathematics. The teacher can use the following to facilitate proper writing about mathematics.

- Allot time for math learning logs

  In learning logs, the teacher guides the students to write the most important vocabulary, concept ideas, procedures, formulas, and examples in their notes.

- Use sentence frames

  Sentence frames help students write mathematics in the proper context. The teacher scaffolds the proper syntax and vocabulary of important mathematical concepts. This process is especially important for English language learners.

Example: The ______ measures ______ inches.

- Allot time for math journals

  In math journals, students have opportunities to explain, reflect, practice, and review mathematical concepts. During this process they expand their mathematical thinking and internalize the concepts.

Teachers can direct students to write:

- about their understanding of how to solve problems
- an explanation of how they solved a problem
- about their level of comprehension on a certain mathematical skill
- predictions of how they think certain problems should be solved
- questions they have for the teacher
- a review of the steps or procedures involved in solving problems
- sample word problems or math-related stories where they can apply the mathematical concepts that they are learning
- a reflection on new concepts learned that day
- a self-assessment of how well they feel they did on homework or a quiz

For elementary school teachers, writing is a core subject that often is neglected when they have to focus on other subjects. These teachers will find that students can gain important skills when they learn to write about mathematical reasoning. Secondary teachers might find that even when students understand mathematical concepts, they may not be able to express their understanding. When these teachers can effectively combine writing instruction and mathematical-reasoning instruction, they are building effective communication about mathematical concepts.

Integrating Mathematics and Science

Mathematics and science are disciplines that can often be taught together quite naturally. When teachers use real-life science experiments to practice mathematical skills, students are able to understand the science and

mathematics more completely because they develop ownership in their experiments. Learning is reinforced and abstract concepts become meaningful, leading to better retention in both content areas. Here are some of the ways that mathematics and science can be easily connected.

Measurement—Science experiments are ideal for applying mathematical measurement concepts to real-life situations. For example, students develop a more complete sense of measuring length when they are handed a ruler and a wiggly worm to measure.
Geometry—The natural world is bursting with shapes to explore. Taking the geometry lesson into life science is a natural way for students to explore geometric concepts.
Analyze scientific experiment data using mathematical formulas—Often, students have a hard time understanding how the formulas they are learning in mathematics relate to the real world. If they are given a chance to apply them to problems they are solving in experiments, it becomes more meaningful.
Finding patterns in naturally occurring scientific events—By exploring real-life patterns that occur in nature, the teacher validates patterns studied in mathematics class.
Algebra formulas in real-life scientific situations—Students often struggle with algebraic concepts. If they are able to see algebra formulas "in action" while conducting experiments, they will have better comprehension of the concepts.
Probability—Both mathematics and science explore the nature of probability. As students conduct experiments, they predict, test, and record the results, providing real-life experience with the concept of probability.

Integrating Mathematics and Social Studies

There are many ways to integrate mathematical concepts with social studies and history. Social studies makes students able to connect with the past, learn about places and people, and apply this learning to the present. Students can use mathematical methods to analyze historical and social patterns. When teachers successfully integrate different content-area concepts, students can more successfully retain information and make connections within their own learning.

Word Problems

The most obvious and perhaps easiest way to integrate social studies with mathematics is to create word problems that combine social studies topics with mathematical concepts that the students are learning. To make sense of mathematical concepts, students often need to apply them to real-world questions.

Here is an example of how to take two Mid-continent Research for Education and Learning (McREL) standard objectives and combine them to create a word problem.

Grades 3–5 Mathematics Standard: Standard 3: Uses basic and advanced procedures while performing the processes of computation

Level II, Benchmark 1: Multiplies and divides whole numbers

Knowledge/Skill Statements: 2. Multiplies whole numbers; 4. Divides whole numbers

Grades 3–5 History Standard: Standard 3: Understands the people, events, problems, and ideas that were significant in creating the history of their states

Topics: 1. Explorers and settlers of the state and region; 2. Cultural diffusion, adaptation, and interaction

Level II, Benchmark 3: Understands the interactions that occurred between the Native Americans or Hawaiians and the first European, African, and Asian-Pacific explorers and settlers in the state or region

Example Problem: In 1850, there were 92,000 people who lived in California, including Native Americans and early settlers. In 1900, there were at least 12 times that number of new arrivals. About how many arrivals had come to California in those 50 years?

Connections to Historical Dates

It is often difficult for students to fully understand the concept of the past. Years and dates have little real meaning. Young children are still developmentally processing the concepts of past, present, and future. **Students of all ages need multiple opportunities to process information about different eras in order to begin to understand the idea of history.** Mathematics can help to clarify these concepts.

Example Problem: The Japanese attacked Pearl Harbor on December 7, 1941. If a U.S. Navy Junior Officer was 25 years old on that day and survived the attack, how old is he now?

Connecting to Historical Data

As students study the places and the people that they are learning about in social studies, they can use mathematics to connect these places to their own lives. They can compare ancient civilizations to the size of their present states and use census data to compare the number of people living in one area to the number of people living

in their cities. They can use maps to calculate and measure differences between locations.

Example Problem: On the map, there are 1,300 miles between Boston and Charleston using the King's Highway, an early American trail. It took almost two months for settlers to walk between those places because they could only travel 20–25 miles per day with their wagons. If you were to walk 25 miles from your city, where could you possibly end up?

Mathematics in the Real World

If teachers are stressing simply the "skills and drills" of mathematical concepts without helping students apply these procedures and formulas to real-world situations, they are doing students a great disservice. When students study a mathematical concept as embodied by a real event or place, they understand and remember it. Application of abstract concepts to real life is a powerful instructional tool.

In each of the following examples, note how the real-life problems push students to broaden their mathematical comprehension.

Primary Skill: Addition

Additional Skill: Classification

How many boys are in the class? How many girls are in the class? How many total students are in the class?

$$13 + 14 = 27$$

Primary Skill: Division

Additional Skills: Multiplication, subtraction, and order of operation

Jon's mom brought 43 cookies to class. There are 25 students in the class. If everyone tries to take 2 cookies each, how many students wouldn't be able to have a second cookie?

$$43 \div 2 = 21 \text{ R1}$$
25 – 21 = 4 students who don't get 2 cookies

How many cookies would we need for everyone to be able to have 3 cookies each?

25 x 3 = 75 cookies

What if the teacher wanted a cookie, as well?

25 x 3 + 1 = 76 cookies

Primary Skill: Perimeter

Additional Skills: Multiplication and money

As a fund-raiser, our class walks laps around the block. Gates Street is 165 feet long. Johnson Avenue is 100 feet long. Brooks Boulevard is 200 feet long. Carter Street is 143 feet long. If we circle around the block 1 time, how far have we gone?

165 ft. + 100 ft. + 200 ft. + 143 ft. = 608 ft.

If you get a pledge for $3.00 for each lap, how many times would you have to go to earn $21.00?

$21.00 ÷ $3.00 = 7 laps

How far will you have walked?

608 ft. x 7 = 4,256 ft.

Integrating Mathematics and Technology

Mathematics and technology are naturally associated, but have not always been effectively integrated into the classroom. Students may have been allowed to play computer games that incorporate mathematics drills, but then would be forbidden to "cheat" on activities or tests by using calculators. Now educators understand that technology, such as computers and calculators, can improve students' education (Dean and Florian, 2001). Rather than using computers as drill-practice machines, teachers now encourage students to use spreadsheets, online applets, and more to explore, apply, and display mathematical learning. Students can receive online mathematics tutoring and homework help from a number of websites. Computers and calculators have become tools that students utilize in order to research, gather data, organize their notes on mathematical concepts, and analyze real-world problems and situations.

Using technology can provide students with opportunities to develop and use their mathematical higher-level thinking skills to solve problems that are relevant to their daily lives.

Differentiation within Technology-Embedded Mathematics

Inevitably, teachers find a range of mathematical and technical competence in any group of students. When teachers incorporate technology into mathematics lessons, there are various ways to differentiate instruction in order to meet the needs of all students.

- Use varied grouping strategies when assigning group mathematics projects.
- Vary the level of support given to students as they use computers and calculators to solve problems and display data.

- Be flexible when setting time limits for work that requires technology tools.
- Allow students to use extra technology tools for scaffolding purposes, when necessary.
- Use multiple technological representations of mathematical concepts.

Using Spreadsheets in a Mathematics Classroom

Spreadsheet programs such as Miscrosoft Excel® can be valuable resources for the mathematics teacher. There are infinite possibilities for use. Unlike word processors, spreadsheets allow **the organization and manipulation of data in specified column and row locations**. In a spreadsheet, there are many ways to display and apply formula functions to numbers. While teachers of all subjects may use spreadsheets to keep records of grades, classroom budgets, attendance, and checklists, the mathematics teacher can also use them for instruction.

	A	B
1		
2		
3		
4		

Students can use spreadsheet programs to:

- manipulate data related to classroom studies
- build analytical, mathematical, interpretive, and technical skills
- compute mathematical formulas
- show mathematical patterns
- create graphs that show algebra and trigonometry relations and functions

Creating, Reading, and Interpreting Graph Information

Many state content standards require that students learn to create and interpret these types of graphs. Students can use spreadsheets to predict changes in numbers, build different kinds of graphs, and compare and manipulate many kinds of data.

Formulaic Manipulation of Data

With a spreadsheet, students can learn about, apply, and experiment with many different orders of operation.

Some of the functions include:

- addition, subtraction, division, and multiplication
- displaying a value as currency, percent, or decimal
- calculating means and averages
- calendar formulas and conversions
- applying specific algebraic functions to data (such as $x + 2$ in a certain column)
- calculating running balances
- creating multiplication tables
- calculating percentages and decimals
- converting measurements
- rounding numbers

Money

Spreadsheets can be used to study a wide variety of financial topics. Students can use spreadsheets to create and manage budgets. Spreadsheets will display negative balances in an alternate color, such as red. Students can create formulas, enter many types of financial data, and keep long-term records.

Research Products

Students can use spreadsheets to display information that they have researched. For example, they might research nutritional recommendations, create charts to display the information, conduct surveys of the actual nutritional intakes of their classmates, and create comparison charts. Students can also use spreadsheets to organize science and mathematical data in the same way.

Using Applets in a Mathematics Classroom

What Are Applets?

Applets are software-component programs that perform precise functions on web pages or in computer programs. These programs can be used as **tools** (such as a calculator) or for **entertainment value** (such as a small pop-up game to play) for the people who visit the web pages or use the computer programs. However, unlike a complete computer program, applets do not run independently. A spreadsheet program might contain an applet that functions as a calculator or graph maker. A word-processing program might contain an applet text editor or synonym finder. An email manager might access an applet that displays video or photograph files. An applet is usually designed to perform a single action and have built-in restrictions that prevent it from harming the user's computer.

There are many educational websites that provide free applets for download. Some are simply for fun, such as making an image wave back and forth, and others have more practical, academic applications such as creating graphs. There are many websites that provide instructions on selecting, installing, and using applets.

Strategies for Using Applets in a Mathematics Classroom

A mathematics classroom should provide access to various websites that offer applet tools that students can use as they are processing and learning mathematical concepts. Some of these sites may have **virtual manipulatives** applets. A teacher and students might find algebra tiles to maneuver, logic puzzles to solve, money to make change with, or blank bar graphs, line graphs, and pie graphs that allow students to enter and display data. These applet tools assist differentiation in the mathematics classroom because students can employ the applet that best matches the skill they are building and progress at their own speed.

Computer Lab Use

In order to use applet tools in a mathematics classroom, students ideally have access to their own computers. In a computer lab, the teacher can model their use on a large display from a master computer, and then students can practice at their own pace at individual computers as the teacher monitors their progress.

Classroom Use

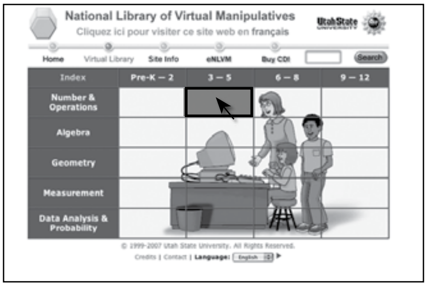
If a computer lab is not available, a teacher can use an overhead projector or other display to model the various applet tools to the whole class during the mathematics lesson. Certain applets can then be designated for individual use during practice time for pairs of students.

Here are three sample applets for elementary, middle school, and high school levels of mathematics. They suggest the endless possibilities for mathematical skills and concept practice that applet tools offer to students.

Examples of Applet Use in an Elementary Classroom

Enter the website for The National Library of Virtual Manipulatives from Utah State University at **http://nlvm.usu.edu/en/nav/vlibrary.html**

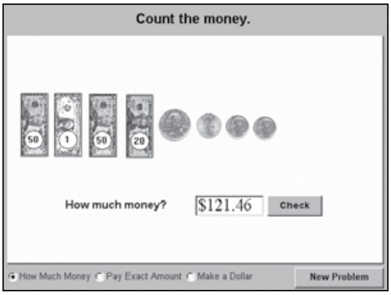
Click on the chart **Numbers and Operations**, **Grades 3–5**



Find and click **Money** in the chart provided.

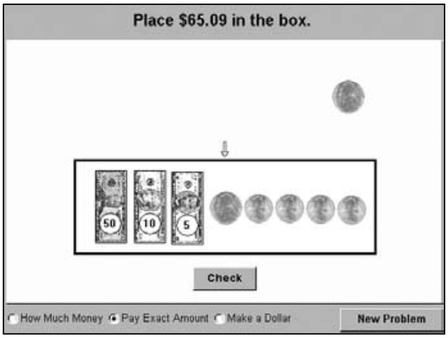
Teach the students how to practice the three activities.

How Much Money?



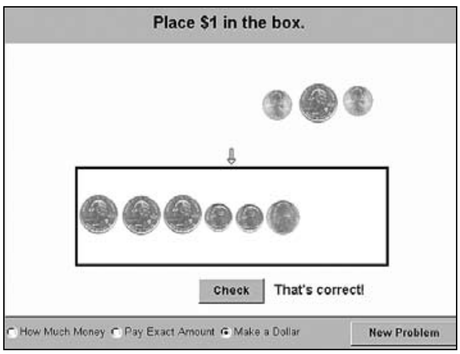
The applet shows a set of bills and coins. The students type in the amount of money shown and click on the word *check* to find out if they are correct. Then they click on the words *New Problem* to request a new problem.

Pay Exact Amount



The applet shows sets of bills and coins. There is an amount of money shown above the box. The students click and drag the correct set of bills and coins to the box. Then they click on the word *check* to find out if they are correct. Then they click on the words *New Problem* to request a new problem.

Make a Dollar

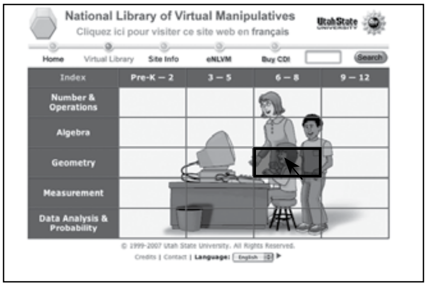


The applet shows a set of coins. The students must drag the correct coins to the box to make a dollar and click on the word *check* to find out if they are correct. They click on the words *New Problem* to request a new problem.

Example of Applet Use in a Middle School Classroom

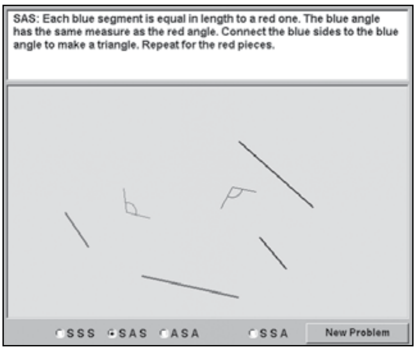
Enter the website for The National Library of Virtual Manipulatives from Utah State University at **http://nlvm.usu.edu/en/nav/vlibrary.html**

Click on the chart **Geometry**, **Grades 6–8.**

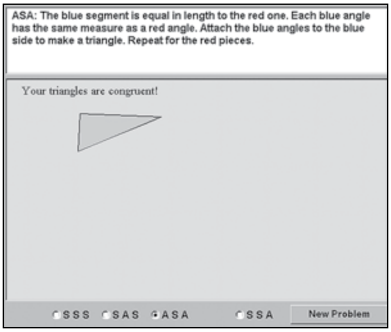


Find and click **Congruent Triangles** from the chart provided.

Teach the students how to practice making congruent triangles using different combinations of sides and angles.



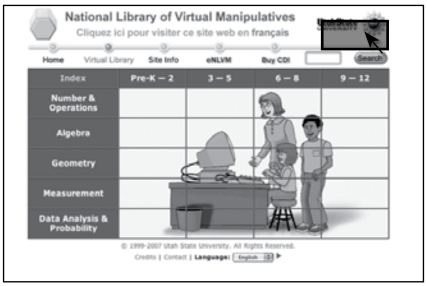
Instructions for each combination are given along the top of the applet. When the blue and red triangles are matched together, they turn green. A message appears if they are congruent.



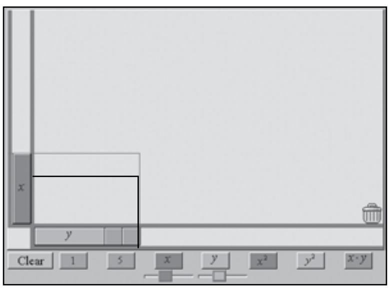
Example of Applet Use in a High School Classroom

Enter the website for The National Library of Virtual Manipulatives from Utah State University at **http://nlvm.usu.edu/en/nav/vlibrary.html**

Click on the chart **Algebra**, **Grades 9–12.**



Find and click **Algebra Tiles** from the chart provided.



Teach the students how to manipulate the virtual *x*- and *y*-axes and tiles in order to solve algebra problems.

Using Graphing Calculators in a Mathematics Classroom

The graphing calculator can be an important tool in all areas of mathematics.

Graphing calculators can help elementary through high school students use higher-level thinking and apply mathematical concepts to specific problems.

Graphing calculators alone cannot achieve the goals that educators have for student success. Teachers need to show students how to use graphing calculators as a tool for mathematical-concept comprehension and application.

A study by Grouws and Cebulla (2000) notes that "teachers ask more high-level questions when calculators are present, and students become more actively involved through asking questions, conjecturing, and exploring when they use calculators."

Classroom Management of Calculator Use

Many teachers dread class-wide calculator use because it requires careful management. However, with proper organization, calculators can allow teachers to spend even more time developing mathematical understanding, reasoning, number sense, and application.

1. All the calculators should be numbered before students receive them. Each student should be assigned a calculator number. This way, the teacher can easily keep track of whether calculators are damaged or not returned.
2. All class sets of calculators should be distinctly marked with bright paint or permanent marker on the outside cover so that they are easily identifiable by teachers, administrators, or other students if they are taken out of the classroom.

3. The calculators should be stored in plastic shoeboxes or in an over-the-door shoe rack with the corresponding numbers printed on the slots.
4. Sufficient class time should be allotted for the organized retrieval and return of the calculators. If students will need the graphing calculators at the beginning of the class period, instructions should be displayed as the students arrive.
5. The teacher should always check for return and damage after calculators have been used. The teacher can use a check-off list to quickly note which numbers are returned without damage.

Teaching Graphing Calculator Skills

Students can learn, reinforce, and review mathematical concepts using graphing calculators. Practice with the calculators will solidify students' understanding of such concepts as number sense, algebraic thinking, data analysis, spatial reasoning, problem solving, and units of measurement (Grouws and Cebulla, 2000). Teachers must integrate graphing calculators into their lessons with this principle in mind.

1. Once students receive the calculators, the teacher should demonstrate keying and data entry skills.
2. To teach a skill, the teacher should request that the students locate the keys and functions on the calculator. The teacher should familiarize students with the menus and screens that these keys and functions access. Students need time to ask questions and explore the functions.
3. If multiple steps are needed to complete the activity, the teacher can list the steps on a poster display, overhead projector, or focus projector so that students can reference the steps during the lesson.

4. The teacher can designate students who are more familiar with calculator use to assist those who are not.
5. To keep students on task, they should expect to share their work with partners, the teacher, or the whole class at any time.

The "I Do, We Do, You Do" Approach

> Graphing calculators are capable of providing multiple representations of mathematical concepts. By building tables, tracing along curves, and zooming in on critical points, students may be able to process information in a more varied and meaningful way. (Smith, 1998)

Graphing calculators can build on conceptual understanding by allowing students to practice numerous representations of concepts and experiences in a way that is not possible by using paper and pencil alone. As a result of these methods, teachers are able to engage students more effectively by addressing different learning styles and developing understanding that leads to higher-level thinking. Teachers don't often associate the use of graphing calculators with the conceptual process. Graphing calculator activities can engage students in building conceptual understanding while giving the practice necessary for procedural proficiency in calculator use. As students move through each phase of learning, they are exposed to a concept or skill numerous times by utilizing the following approach in each lesson.

1. **I do**—The teacher models the complete procedure using a display product such as a computer program or projector.
2. **We do**—While the teacher repeats each step, the students engage in the step as a class.
3. **You do**—The students practice the complete procedure independently.

Post-Reading Reflection

1. Why is it important to integrate mathematics across the content areas?

2. Describe how students can use a math journal in each of the following content areas: reading, writing, social studies, science, and mathematics.

3. Explore the National Library of Virtual Manipulatives site. Find an applet that goes with a lesson you need to teach this year. Write about how you will use the applet in that lesson.

Assessing Students

> Assessments are learning opportunities for students and for teachers. Students can learn more mathematics, and teachers can learn more about their students—and sometimes more mathematics, as well—from all types of assessments. (Long, 2000)

Aligning Assessment with Instruction

The goal of standards-based instruction is to have a set of concepts that all students can master by the end of the year. In order to accomplish that goal, teachers must assess their students appropriately throughout the year. Assessments can take many forms and should give teachers the information they need to make informed decisions about what to teach and how to teach it.

In order for teachers to appropriately use the information gathered from assessments, the assessments must align closely with the mathematical concept being taught. If not, teachers will not know whether students truly understand the concept.

Teachers must consider several things when creating new assessments or reviewing previously created assessments to use with their students.

- The assessment should cover material that was taught.
- The mathematics should be relevant and engaging to the students.
- The expectations for the finished product should be clear.
- The assessment should clearly show students' mathematical knowledge, understanding, and thinking processes.
- The activity used for assessment should have a clear purpose for either formal or informal assessment.

When assessment aligns with instruction, both teachers and students benefit. Students are more likely to gain a deep understanding of the curriculum when instruction is focused and they are assessed on what they are taught. Instruction-aligned assessments are also time effective for teachers because they monitor learning and can be integrated into daily instruction and classroom activities.

When to Assess

Assessment is a long-term, ongoing process. Teachers can gain insight into students' learning through informal observations, listening to groups communicate, activities and projects, and formal tests and quizzes.

There are several points throughout a lesson where useful assessment can be made. Depending on the results, teachers can decide if they should continue with the lesson as planned or change gears if student understanding is low. Those points of a lesson include:

- after accessing prior knowledge
- during guided practice
- during independent practice

Informal Assessments

Informal assessment is a valuable way for teachers to get a quick understanding of how students are progressing in understanding a mathematical concept. However, given that these assessments are informal, teachers often do not record them and have no evidence that indicates student growth. It is important to have easy methods of recording informal assessments to use for grading reports, parent conferences, and to show administrators.

Recording Methods

Maximizing instructional time is an important skill that teachers need in order to cover the necessary material throughout the year. Although assessment is one of the most important aspects of education, often teachers do not allot appropriate time for meaningful assessment. Teachers can use easy methods for recording informal assessments that do not take additional class time to complete. Using a clipboard or small binder is a simple way to allow for portability of the recording sheets.

- Use a blank grid to record notes for each student during a discussion, pair activity, or small-group work.
- Design a check-off sheet with students' names down the side column and a list of several objectives or concepts being taught in the lesson

across the top. Give students a check in each area in which you observe them showing mastery throughout the lesson.

- Post a schedule of students who will individually meet with the teacher each day of the week. Have an activity or quick task they can complete while at their meeting. Use a data sheet to record any misconceptions and observations while each student is working.
- Use a general rubric to gauge student understanding. Walk around the room to observe and record understanding for several students each day to see how they are progressing.

Check for Understanding

Another way to maximize instructional time is to check often for understanding during the lesson. When this is done, the teacher can use this knowledge to decide whether to proceed with further lesson concepts, repeat instruction for some lesson concepts, offer more practice with the concept, or skip a portion of the lesson that the students already understand. The teacher may use the knowledge to differentiate instruction by deciding how many practice problems and activities are necessary or by allowing some students to work independently while the teacher works with a small group of students who need more instruction. Overall, checking for understanding guides the teacher in how to adjust instruction so that the lessons directly meet students' instructional needs (Wiliam, 2007).

Strategies for Checking for Understanding

Goal	Strategy	Summary of Strategy
Students are comfortable with the pace of the lesson. They are comfortable with the concepts they are learning.	Display thumb signal	The teacher asks the students to show a signal under their chins (for privacy). They place their thumbs up, down, or wavering in the middle to demonstrate the following: "Yes, I totally understand this," "No, I do not understand this," or "I think I understand this."
The students understand how to solve an example problem.	Pair sharing	The students each solve a problem and share it with their partner. When both partners agree on the solution, they show the teacher a "thumbs up" signal.
The students can solve a problem correctly and get the right answer.	Using whiteboards	The students complete a problem on whiteboards and show their answers to the teacher. The teacher can quickly gauge whether students are solving the problems appropriately and can then make lesson decisions accordingly.
The students understand the concept well.	Fast writing assignment	The students can quickly write two to three sentences explaining the concept or sequencing the procedure for solving a problem.

Formal Assessments

Tests and Quizzes

Written tests and quizzes are often the most common type of formal assessment used in classrooms. These types of assessments are useful when teachers want to see what a student understands within a specific mathematical concept. They can also be helpful to gauge

long-term understanding of concepts and how students are applying the mathematical skills learned throughout the year. There are many ways in which tests and quizzes can be utilized in mathematics.

- Teachers may use short diagnostic tests when beginning a unit of study to discover students' prior knowledge.
- Teachers may use a small quiz as a quick-check for understanding within a unit of study to see if students are making progress.
- Teachers may use a unit test that combines several mathematical concepts as a culmination to the unit.
- Teachers may allow students to create their own tests and answer keys as a means for checking their understanding of a concept.

Rubrics

The Effectiveness of Using Rubrics

The use of rubrics can be an effective assessment strategy in a mathematics classroom. A teacher may want to create general rubrics to aid in the process of using assessments to direct further instruction. With rubrics, teachers can create and convey realistic expectations for student work as well as their own teaching strategies and lesson planning in order to refine their objectives for the class.

When a teacher communicates clear assignment directions and then provides a rubric that delineates the expectations of the completed assignment, the students are supplied with all the necessary information they need in order to be successful with an assignment. Rubrics can be especially useful for assignments where students have to explain their reasoning or their comprehension

of a particular lesson concept. In this case, the teacher could create one general rubric that could be used many times throughout the year.

A well-written rubric contains the objectives that the student will achieve when he or she completes the assignment and performance indicators that measure his or her progress. Teachers need to teach students how to examine rubrics before beginning assignments, check their work throughout the assignments, and finally use the rubric guidelines to evaluate the completeness of their work once they are finished. Rubrics allow teachers to define what makes an assignment complete before students begin work, so they can anticipate how they will be evaluated and accept their final grades.

Creating a Mathematics Rubric

1. Before creating a mathematics rubric, define the assignment's objectives. State standards are a great place to begin when creating a rubric because they show exactly what needs to be mastered.
2. Generate a list of the concepts the students are expected to master as well as another list of ways they can show that they comprehend the material.
3. Highlight the most important items on both lists.
4. Describe the performance criteria in detail. These should clarify each phase of the work.

It is important for teachers to decide how many criteria to include. The more detailed the rubric, the more involved the scoring process. Many teachers find that four to five items are ideal. Too few, and the teacher will be unable to effectively evaluate the performance of students with varied learning abilities. Too many, and the rubric will be too extensive and time consuming to use. The rubric should be easily comprehensible for students so they can evaluate their own performances.

A General Overview of Rubric Guidelines

Lowest Level	Middle Level	Highest Level
• Student shows minimal comprehension of the concept and does not complete the assignment or task. • Many of the assignment's or task's required components are missing. • The student must continue to work on the assignment or task in order to comprehend the concept.	• Average achievement; lowest acceptable score. • The student shows basic understanding of the concept but not complete mastery.	• Best possible completion of the assignment or task. • A score at this level demonstrates total comprehension of the concept.

Once the teacher has identified the lowest, middle, and highest levels of performance, he or she can fill in the intermediate levels.

Sample rubrics can be found on pages 177 and 178. There is also a *General Rubric* (page 179) that shows general criteria for various steps of an assignment, including completion of problems, correct calculations, answers that relate to the topic, and logical reasoning.

Two-Point Rubric

Points	Criteria
2	• The solution is correct and the student has demonstrated a thorough understanding of the concept and procedure. • The task has been fully completed using sound mathematical methods. • The response may contain minor flaws, but it is thorough and the understanding is evident.
1	• The response is only partially correct. • The solution may be correct, but the response demonstrates only a partial understanding of underlying mathematical concepts or procedures. • Or, the solution is wrong, but the student demonstrates understandng of the concept of procedures.
0	• The solution is incorrect. • The response is incomprehensible and/or demonstrates no understanding of the concept.

Rubric adapted from *FCAT 2004 Sample Test Materials* (2003, Florida Department of Education).

Four-Point Rubric

Points	Criteria
4	• The solution is correct and the student demonstrates a thorough understanding of the concept or procedure. • The task has been fully completed using sound mathematical methods. • The response may contain minor flaws, but it is thorough and the understanding is evident.
3	• The solution demonstrates an understanding of mathematical concepts or procedures involved in the task. • The response is mostly correct but contains small errors in execution of mathematical procedures or demonstration of conceptual understanding.
2	• The response is only partially correct. • The solution or method for solving the problem may be correct, but the response demonstrates only a partial understanding of underlying mathematical concepts or procedures. • Errors show misunderstanding of parts of the task or faulty conclusions.
1	• The response shows limited understanding of mathematical concepts and procedures. • The response is incomplete and the parts of the problem that have been solved and/or explained contain serious flaws or incomplete conclusions.
0	• The solution is completely incorrect. • The response is incomprehensible and/or demonstrates no understanding of the concept.

Rubric adapted from *FCAT 2004 Sample Test materials* (2003, Florida Department of Education).

General Rubric

Directions: This rubric includes general criteria for grading multistep assignments that involve written explanations to questions. In each of the columns, specify criteria and explain how they relate to the activity and the levels of performance that can be achieved. To evaluate an activity, circle a level of performance for each criterion and assign a number of points. Total the points and record them.

Criteria	Level I (0–4 pts.)	Level II (5–8 pts.)	Level III (9–10 pts.)	Self-Score	Peer Score	Teacher Score
Steps in the activity have been completed.						
Question(s) have been answered. Responses relate to the questions being asked.						
Calculations are shown and/or explained.						
Ideas are supported with logical reasoning and/or evidence.						

Data-Driven Instruction

Data-driven instruction refers to the process of designing curriculum and instructional strategies to match data from student assessments. This data can be collected from various daily activities such as student-teacher interaction and observations, guided and independent practice, and formal and informal assessments. The key to data-driven instruction lies in gathering and interpreting the data and understanding how to use it.

> Data analysis is meaningless if it does not result in meaningful instructional change. Data-driven educators are able to use summative and formative assessment data together to implement strategic, targeted, focused instructional interventions to improve student learning. These interventions should be aligned with state standards and district curricula as well as content-specific, developmentally appropriate best practices. (McLeod, 2005)

Once the data is collected, it needs to be analyzed. Data can be analyzed by student or by skill/concept/standard. When looking at data from a particular student, it is important to see what skills they are not mastering and any commonalities those skills possess. For example, if the student is having a difficult time multiplying and dividing, he or she is probably weak in addition and subtraction because those are the skills built from a clear understanding of multiplication and division. Identifying the mathematical concepts with which the student is struggling will guide future intervention for that particular child. When looking at data by concept, skill, or standard, a teacher can see where a group of students is having difficulty. Depending on the number of students having difficulty, a teacher can create an intervention with just that particular group of students, or reteach the concept to the whole class (Dean and Florian, 2001).

Data analysis is most efficient when the proper tools are used for organizing the data. The tables in the following pages will aid in analyzing class data.

Assessment Item Analysis

This data organizer (page 182) can be used to evaluate how each student answered particular questions on an assessment of multiple skills. If students did poorly on one type of question, it may show that a skill has not been mastered or that they do not have the underlying skills necessary to solve that problem. This data can help identify skills and concepts that need to be retaught. If a teacher is using a rubric to evaluate a performance-based or informal task, point values should be assigned to the rubric and then the teacher should utilize this chart as instructed.

To complete the chart:

- Correct students' tests and determine the number of points each item is worth.
- Record the students' names in the first column and the item numbers in the first row of the table.
- Write the number of points a student received on each item underneath the corresponding item number. For example, if each item on a test is worth one point, record a 1 for a correct answer and a 0 for an incorrect answer.
- At the bottom of the table, record the total points earned in the class and the percentage correct for each item. First, add the total points that a class earned on an item, collectively, and divide it by the total possible points that the class could have earned. Then, multiply the decimal by 100.

Assessment Item Analysis

Unit:____

Assessment Title: ____

	Item Numbers															
Students' Names																
Total Points																
Percentage Correct																

Assessment Item Analysis by Standard

This data organizer (page 184) can be used to gain insight into the progress being made for a given standard by a particular student or the whole class. If a teacher is using a rubric to evaluate a performance-based or informal task, point values should be assigned to the rubric and then the teacher should utilize this chart as instructed.

To complete this chart:

- Write the students' names in the first column of the table.
- Record the number of the standards assessed in the diagonal columns.
- In the row below the standards, record the item number(s) from the assessment that correlate to each standard.
- In the second row, record the total number of possible points that students could earn on those items.
- After grading the assessment, find the total number of points students earned for each standard by counting the points for the items designated for that standard.
- After analyzing the assessment by standard, determine the percent of mastery per standard. First, calculate the total number of possible points that students could earn on a given standard. Then, add up the total number of points that they actually earned. Finally, divide the total number of points the students earned collectively by the total number of points they could have earned collectively and multiply the decimal by 100.

Assessment Item Analysis by Standard

Unit: ______________________________

Assessment Title: ______________________________

Standards										
Item Numbers										
Points Possible per Standard										
Total Points										
Percentage of Points Earned										

Students' Names

Using the Data Collected

Data by itself is not inherently useful. It is not until teachers put the data into a workable document and analyze the results that the data becomes something they can use in the classroom. Teachers can use the data to communicate with students as well as to identify and address student misconceptions to guide future instruction. Data can be used to convey information to parents, administrators, and the community, as well.

Communicating with Students

It is important to show students data in order to help them understand their progress throughout the year. At times, letter grades can become daunting and pulling up a grade seems impossible. When students have data that measures their performances on a small task, they can see specific ways to improve their mathematical understanding. This gives them workable, meaningful goals. When students gain mastery of a mathematical skill, it increases their confidence and motivation to continue to achieve in other areas of mathematics (Wiliam, 2007).

Strategies for Identifying Student Misconceptions

It is a fact that there are many students who fail or struggle in mathematics classes. Furthermore, students do not always understand why they are having problems in a mathematics class. They might realize that they are struggling, but feel powerless to "catch up" once they fall behind. They might claim that they "do not like math" when they are really hiding behind the reality that they do not understand many mathematical concepts. Sometimes students feel discouraged and react angrily in efforts to hide their senses of failure. While a teacher might find some students who actively seek a path for understanding, the teacher will also have students who do not know how to do so.

Here are some practical strategies for identifying student misconceptions and understanding.

1. The teacher uses frequent formal and informal assessments to check for understanding during lessons.
2. The teacher carefully watches assessments of individual students for evidence of mastery or confusion.
3. When the results of the whole class are analyzed, the teacher uses these diagnostic tools to help identify which concepts need the most instructional time.
4. The teacher keeps accurate records of student and class progress and uses them to see where students are struggling.
5. The teacher carefully monitors classroom and homework assignments. While these do not always need to be graded, the teacher uses them for signs of how well the students are grasping the contents.
6. The teacher reviews the students' notes taken in class. The teacher can periodically ask students to explain their comprehension of vocabulary, procedures, or concepts in order to address student misconceptions in further lessons or decide if reteaching the lesson is appropriate.
7. The teacher uses rubrics. With a rubric, the teacher can identify the skills evident or lacking in student work. The teacher can share rubrics with students, or have the students identify the rubric scores on their own papers or the papers of peers, with the names covered. This will help students focus on the most important concepts of a lesson.

Addressing Student Misconceptions

Once the teacher has given the assessments, graded them, collected and analyzed the data, and identified the areas in which the students are faltering, the teacher chooses an action plan for addressing student misconceptions and reteaching materials. It is always more effective to change the strategies and instructional plans when reteaching a concept that the students did not understand the first time.

Suggestions for Reteaching the Concepts
The teacher may not have taught all the components of the lesson plan. The teacher can decide to **teach the skipped components** to reteach the concept.
The teacher can **choose a strategy from the differentiation chart** to use for reteaching the concept. The teacher should choose the appropriate level from the chart in order to pick an effective strategy for each group of students.
The teacher can allow students **opportunities to talk about the concept** with partners or within small groups.
The teacher can **choose an activity** such as a game or a project that further enhances students' understanding but presents the information in a new way.
The teacher can allow those who successfully mastered the content to review different application activities while **meeting separately with a small group** of those who did not master the content.
The teacher can **review the vocabulary with activities** if the teacher suspects that the students still do not have the academic vocabulary necessary to comprehend the concepts and practice the skills.

Communicating with Parents

Data can be used as an effective tool in parent-teacher conferences and communications. It allows parents to see specifically where their children are struggling or excelling, and provides useful feedback for ways in which parents can help at home. The data will clarify what is expected of students and where their children are actually performing according to the state standards.

Communicating with Administrators

Data can be used to show administrators how students are progressing throughout the year in relation to mastery of state standards. This will give administrators an idea of how students may perform on any state-mandated, standardized assessments. The data will also indicate if or when during the year to begin an intervention program before or after school. Administrators would also be interested in the data to anticipate which individual students and how many in total will likely be retained at their schools.

Communicating with the Community

In many states and districts, data is a measure of accountability for students, parents, teachers, and administrators. When the data reveals an overall trend in a program, school, or district, it provides valuable information to show what is working and what is not working in the community's schools.

Post-Reading Reflection

1. Why is continual assessment integral to the success of your students?

2. How can you use data to understand what concepts need to be taught in the future?

3. Create a rubric that assesses an informal project or activity that you already use in your classroom.

Conclusions

Developing an Intervention Curriculum

For students struggling in mathematics, it is important to receive some type of intervention as soon as possible so they do not fall behind. According to the Response to Intervention (RTI) method, not all students need the same type of intervention. Tier I students will only need intervention as part of daily classroom mathematics activities. Teachers of these students should be able to complete this type of intervention on their own. Tier II and Tier III students need more help than a classroom teacher can do on his or her own. These students need outside interventions. These interventions can take place in various settings and require thoughtful planning.

A curriculum team should meet frequently to design intervention programs. These meetings need to begin at

the end of the previous year, during the summer, or at the very beginning of the current school year. The sooner decisions are made regarding the programs that will be implemented, the more time there will be to develop the timeline, plan instruction to meet students' needs, and conduct professional development. Decisions that need to be made include:

- determining which type(s) of intervention will be offered
- determining the amount of instructional time for each type of intervention
- deciding the length of each program
- looking at the specific curriculum that has been chosen and creating a timeline for teachers to follow
- assigning and training personnel

Instructional Intervention Settings

The curriculum team can choose between many different types of intervention settings depending on the needs of the students, assessment results, resources available, and time limitations.

Small-Group Instruction

This intervention setting includes but is not limited to pull-out programs for struggling students, special education students, and/or English language learners. In small-group instruction, students receive additional instruction on the mathematical concepts with which they have difficulty, and more concrete practice of basic mathematical skills and targeted standards.

Before-/After-School Program

In before-school and after-school programs, students receive quiet instructional time away from the distrac-

tions of the regular school day. They are not missing any activities or classroom instruction during this time. Students can receive remedial help or prepare for future class lessons through extra review and practice.

Saturday School

Similar to a before-school or after-school program, Saturday school provides a time for students who normally have outside community commitments such as sports practice, religious activities, or volunteer opportunities, to come for additional mathematics instruction. Students tend to arrive to Saturday school focused and ready to learn mathematics because they do not have the other subjects and distractions that are part of the regular school day.

Summer School

A district may offer summer school for students in the summer before they begin a new course, or as a remedial time for students who need to strengthen their understanding of mathematical skills introduced during the previous year. Because summer school is typically scheduled for four to six weeks, students can improve their skills at a slower pace and in a more focused manner.

Keys to Remember When Using an Intervention Program

- Include multiple representations of concepts.
- Allow students to move through the stages of mathematical development (concrete, abstract, application) slowly so that sound connections are made between each step.
- Create an environment in which students feel comfortable asking questions and discussing concepts when they do not understand something.

- Use creative instructional strategies for reteaching.
- Keep open communication among teachers, administrators, students, and parents so that everyone understands what progress is being made and what learning still needs to be developed.
- Help students choose learning goals to keep them motivated throughout the intervention program
- Be positive and encouraging with students at all times.
- Have other relevant, mathematics-based activities or games students can turn to as a break from the intervention curriculum if their frustration levels get too high.

What Should Mathematics Instruction Look Like?

At this point, teachers may be feeling overwhelmed by all the strategies and best practices advocated within this book. It is essential to combine these strategies and make sense of how they work together. This section will show teachers how to put it all together.

Mathematics instruction must adapt to the needs of the students in each classroom. However, there are some general guidelines to follow in every activity and lesson.

1. **Introduce the standards-based mathematical concept.**
    - Make sure the students are engaged and excited.
    - Access students' prior knowledge.
    - Complete a vocabulary-building activity.
2. **Teach the lesson.**
    - Use differentiated instruction.
    - Use manipulatives.

- Begin with concrete concepts and eventually move to the application stage (this will take more than one lesson).

3. **Informally assess understanding.**
    - If there is high understanding, move on to the next step.
    - If there is low understanding, reteach the concept before moving to the next step.
4. **Begin guided practice.**
    - Use differentiated instruction.
    - Use manipulatives.
    - Utilize small groups and peer-assisted learning.
    - Be very involved with students' learning and provide continual feedback to them about their progress.
5. **Informally assess understanding.**
    - If there is high understanding, move on to the next step.
    - If there is low understanding, reteach the concept before moving to the next step.
6. **Begin independent practice.**
    - Use differentiated activities.
    - Use games or activities to reinforce guided practice.
    - Utilize cooperative learning, group work, small groups, or independent work.
    - Intervention time here is key for students who are struggling or need extra scaffolding.
7. **Assess the final concept.**
    - Use formal or informal assessment, depending on the lesson.
    - Use results to decide on future lessons.

Key Strategies for Implementing a Successful Mathematics Program

- Teachers need to learn the mathematics curriculum themselves so they can maximize student understanding and be knowledgeable when students ask questions.
- Teachers should always keep their students in mind when planning curriculum and reassessing their curriculum timelines. If students are not making the progress they need to truly learn the mathematical concepts, adjustments need to be made.
- Teachers need to be prepared for the lessons they are teaching, with all supplies and resources readily available.
- Teachers should always instruct students in the concrete stage of a concept before moving to the abstract and application stages.
- Teachers need to differentiate instruction daily to meet the needs of all students in the class.
- Teachers need to be systematic and organized to utilize assessments effectively and in a timely manner.
- Teachers need to utilize the support of administrators and parents in order to maximize the resources available in the classroom.

Post-Reading Reflection

1. What are three keys to creating a successful mathematics intervention program?

2. How has your idea of mathematics instruction changed after reading this book?

3. Make a list of the top three things you want to change or begin doing in your classroom based on the information you have read.

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